

## Test

1

Total mark

5

1 Choose the correct answer from the given ones :

(3 marks)

1  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$

(a)  $\mathbb{Z}^+$

(b)  $\{0\}$

(c)  $\mathbb{Z}^-$

(d)  $\emptyset$

2 The image of the point A  $(-4, 3)$  by translation  $(-1, -4)$  is  $\dots\dots\dots$

(a)  $(-5, -7)$

(b)  $(-5, -1)$

(c)  $(-7, 3)$

(d)  $(-3, -1)$

3  $|-5| + 5 = \dots\dots\dots$

(a) 0

(b) 5

(c) 10

(d) 15

2 Use the properties of addition in  $\mathbb{Z}$  to find the result of :

$-17 + 19 + 17$  (state the property used in each step).

(2 marks)

## Test

2

Total mark

5

1 Choose the correct answer from the given ones :

(3 marks)

1 The additive inverse of  $|-8|$  is  $\dots\dots\dots$

(a) 8

(b)  $-8$

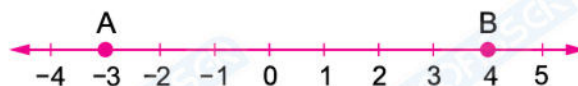
(c) 0

(d) 16

2 On the number line :

The length of  $\overline{AB}$

equals  $\dots\dots\dots$  length units.



(a) 1

(b) 5

(c) 7

(d) 10

3 The greatest negative integer is  $\dots\dots\dots$

(a)  $-1$

(b)  $-10$

(c)  $-100$

(d)  $-1000$

2 In the coordinates plane , find the image of  $\overline{AB}$  , where A  $(2, 3)$

and B  $(-2, 0)$  by translation  $(x + 3, y - 2)$

(2 marks)

## Test

## 3

Total mark

5

## 1 Choose the correct answer from the given ones :

(3 marks)

1  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

- (a)  $\mathbb{Z}$                       (b)  $\mathbb{N}$                       (c)  $\{0\}$                       (d)  $\emptyset$

2 If  $X(-2, 5)$ ,  $Y(-2, -3)$ , then the length of  $\overline{XY} = \dots\dots\dots$  length units.

- (a) 2                      (b) 4                      (c) 8                      (d) 15

3  $3 - |-3| = \dots\dots\dots$

- (a) 0                      (b) 1                      (c) 3                      (d) 6

## 2 Arrange the following numbers in an ascending order :

(2 marks)

$-9$ ,  $17$ ,  $|-9|$ ,  $-15$  and  $16$

## Test

## 4

Total mark

5

## 1 Choose the correct answer from the given ones :

(3 marks)

1  $\{-2, -\frac{1}{2}\} \dots\dots\dots \mathbb{Z}$

- (a)  $\in$                       (b)  $\notin$                       (c)  $\nsubseteq$                       (d)  $\subset$

2  $-4 > \dots\dots\dots$

- (a) 0                      (b)  $-3$                       (c) 4                      (d)  $-5$

3  $|-48| \div |-6| = \dots\dots\dots$

- (a)  $-7$                       (b)  $-9$                       (c) 8                      (d)  $-8$

## 2 In a Cartesian coordinates plane, locate

(2 marks)

the points  $A(0, 4)$ ,  $B(2, 1)$ ,  $C(-2, 1)$ ,  
then find the image of  $\triangle ABC$  by  
translation  $(0, -2)$

## Test

## 5

Total mark

5

**1 Choose the correct answer from the given ones :**

(3 marks)

**1**  $|-7| + 3 \dots\dots\dots |-7 + 3|$

(a)  $>$

(b)  $=$

(c)  $<$

(d)  $\leq$

**2**  $\mathbb{Z}^+ \cup \{0\} = \dots\dots\dots$

(a)  $\mathbb{Z}$

(b)  $\mathbb{Z}^-$

(c)  $\emptyset$

(d)  $\mathbb{N}$

**3** The image of the point (..... , ..... ) by translation $(x - 3, y + 4)$  is  $(-5, -3)$ 

(a)  $(-2, 7)$

(b)  $(-8, 7)$

(c)  $(-2, -7)$

(d)  $(-8, 15)$

**2** Write using the listing method,  
the set of integers between  $-4$  and  $3$ 

(2 marks)

## Answers of Test

1

1 1 c

2 b

3 c

2  $-17 + 19 + 17 = -17 + 17 + 19$  (Commutative property)  
 $= (-17 + 17) + 19$  (Associative property)  
 $= 0 + 19$  (Additive inverse)  
 $= 19$  (Additive identity)

## Answers of Test

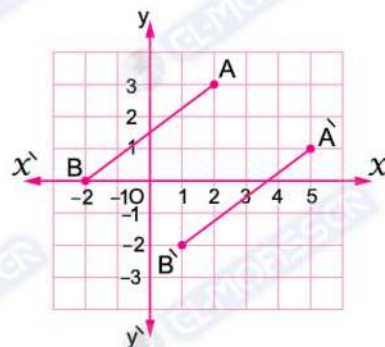
2

1 1 b

2 c

3 a

2  $A(2, 3) \rightarrow \hat{A}(5, 1)$   
 $B(-2, 0) \rightarrow \hat{B}(1, -2)$



## Answers of Test

3

1 1 d

2 c

3 a

2 The order is :  $-15, -9, |-9|, 16$  and  $17$



## Answers of Test

4

1 1 c

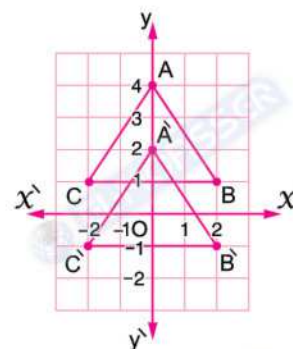
2 d

3 c

2  $A(0, 4) \rightarrow \hat{A}(0, 2)$

$B(2, 1) \rightarrow \hat{B}(2, -1)$

$C(-2, 1) \rightarrow \hat{C}(-2, -1)$



## Answers of Test

5

1 1 a

2 d

3 c

2  $\{-3, -2, -1, 0, 1, 2\}$

**1** Complete each of the following :

[a]  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

[b]  $|-13| = \dots\dots\dots$

[c] The opposite of 7 is  $\dots\dots\dots$

[d]  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$

[e]  $\mathbb{N} \cap \mathbb{Z} = \dots\dots\dots$

5

**2** Put the suitable sign " $\in, \notin, \subset$  or  $\not\subset$ " :

[a]  $-7$    $\mathbb{N}$

[b]  $\frac{9}{3}$    $\mathbb{Z}$

[c]  $\mathbb{Z}^+$    $\mathbb{N}$

[d]  $|-35|$    $\mathbb{N}$

[e]  $\mathbb{N}$    $\mathbb{Z}^-$

5

**3** Write an integer to represent each situation :

[a] A temperature of 2 degrees below zero.

(  $\dots\dots\dots$  )

[b] An increase of L.E. 7

(  $\dots\dots\dots$  )

[c] 9 m. above the sea level.

(  $\dots\dots\dots$  )

[d] A loss of P.T. 80

(  $\dots\dots\dots$  )

[e] A bank deposit of L.E. 95

(  $\dots\dots\dots$  )

5

**4** Choose the correct answer :

[a]  $3 + |-3| = \dots\dots\dots$

( 3 or 4 or 5 or 6 )

[b]  $|25| \div |-5| = \dots\dots\dots$

( 20 or 5 or 10 or 15 )

[c]  $|-11| + 22 = \dots\dots\dots$

( 11 or 22 or 33 or 44 )

[d]  $|3| \times |-6| = \dots\dots\dots$

( 9 or 18 or 20 or 36 )

[e]  $0 \times |-9| = \dots\dots\dots$

( 0 or 9 or 90 or 900 )

5

**5** Represent the following numbers on the number line :

[a]  $-3$

[b]  $|-5|$

[c]  $-|2|$

[d]  $\{7, 8, 9\}$

[e]  $\{-3, -4, -5, -6, \dots\}$

5

1 Put the suitable relation "> , = or <" :

[a]  $-8$    $4$

[b]  $0$    $-2$

[c]  $5$    $|-5|$

[d]  $-3$    $-5$

[e]  $|-9|$    $-|-10|$

5

2 [a] Arrange the following numbers in an ascending order :

$-6$  ,  $15$  ,  $0$  ,  $|-9|$  and  $-18$

[b] Arrange the following numbers in a descending order :

$-9$  ,  $17$  ,  $|-9|$  ,  $-15$  and  $16$

5

3 Choose the correct answer :

[a] The smallest positive integer is ..... ( 0 or 1 or 2 or 3 )

[b]  $\mathbb{Z} - \mathbb{Z}^- = \dots\dots\dots$  (  $\mathbb{Z}^+$  or  $\mathbb{Z}$  or  $\mathbb{N}$  or  $\{0\}$  )

[c]  $|-12| + |-21| = \dots\dots\dots$  ( 22 or 18 or 30 or 33 )

[d] The number ..... is neither positive nor negative.

( -1 or 0 or 1 or 2 )

[e] The opposite of  $|-3|$  is ..... ( -3 or 3 or 4 or -4 )

5

4 Write :

[a] The previous integer and the next integer of  $-27$

[b] The integers between the two integers  $-5$  and  $3$

4

5 Write using the listing method each of the following sets :

[a] The set of integers greater than  $-4$

[b] The set of integers smaller than  $-1$

[c] The set of non-negative integers.

[d] The set of integers smaller than  $5$  and greater than  $-6$

6

**1** Find the result of each of the following :

**[a]**  $(-7) + 2 = \dots\dots\dots$

**[b]**  $(-4) + (-5) = \dots\dots\dots$

**[c]**  $14 - 27 = \dots\dots\dots$

**[d]**  $16 - (-3) = \dots\dots\dots$

**[e]**  $12 + (-12) = \dots\dots\dots$

5

**2** Write the property of addition in  $\mathbb{Z}$  in each of the following :

**[a]**  $(-5 + 6) + 9 = -5 + (6 + 9)$  ( ..... )

**[b]**  $(-8) + 7 = 7 + (-8)$  ( ..... )

**[c]**  $-11 + 0 = -11$  ( ..... )

**[d]**  $14 + (-14) = 0$  ( ..... )

4

**3** Choose the correct answer :

**[a]**  $4 + (-7) - 2 = \dots\dots\dots$  (  $-4$  **or**  $-5$  **or**  $-6$  **or**  $-7$  )

**[b]**  $-8 + 5 + 8 = \dots\dots\dots$  (  $5$  **or**  $6$  **or**  $4$  **or**  $8$  )

**[c]**  $|-7| + \dots\dots\dots = 0$  (  $0$  **or**  $7$  **or**  $-7$  **or**  $14$  )

**[d]**  $4 + \dots\dots\dots = -1$  (  $5$  **or**  $4$  **or**  $-4$  **or**  $-5$  )

**[e]** The additive inverse of  $(-5)$  is ..... (  $0$  **or**  $5$  **or**  $-5$  **or**  $50$  )

5

**4** Use the properties of addition in  $\mathbb{Z}$  to find :

**[a]**  $-15 + 29 + 15$

**[b]**  $55 + (-255) + 45 + 255$

6

**5** Arrange each of the following in an ascending order :

**[a]**  $|-3|$  ,  $5$  ,  $-3$  ,  $0$  and  $4$

**[b]**  $1$  ,  $11$  ,  $-1$  and  $-11$

5



**1** Determine the position of each of the following points

A (1, -1), B (4, -1) and C (4, 5), then find :

**[a]** The length of each of  $\overline{AB}$  and  $\overline{BC}$

**[b]** The type of the triangle ABC with respect to its side lengths and its angles.

**[c]** The area of the triangle ABC

5

**2** In the opposite figure :

ABCD is a rhombus, choose the correct answer :

**[a]** A (....., .....)

((1, -1) or (-1, 1) or (0, 1) or (-1, 0))

**[b]** D (....., .....)

((1, 4) or (4, 1) or (-2, 4) or (1, -2))

**[c]** The length of  $\overline{AC}$  = ..... units.

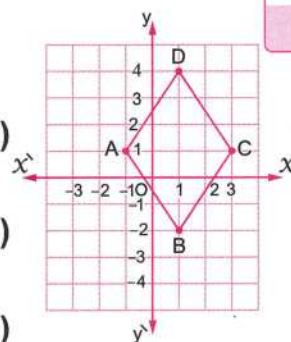
(3 or 4 or 5 or 6)

**[d]** The length of  $\overline{BD}$  = ..... units.

(3 or 4 or 5 or 6)

**[e]** The surface area of the rhombus ABCD = ..... square units.

(4 or 6 or 12 or 24)



5

**3** Determine the positions of X (-2, 2), Y (-2, -3), Z (3, -3) and L (3, 2), then find :

**[a]** The name of the shape XYZL

**[b]** The perimeter and the area of the shape XYZL

**[c]** The number of axes of symmetry for the shape XYZL

5

**4** Determine the positions of L (-2, -1), M (1, -1), N (1, 3) and P (-2, 3), then find :

**[a]** The length of each of  $\overline{LP}$  and  $\overline{PN}$

**[b]** The perimeter and the area of the shape LMNP

5

**5** On a square lattice, draw  $\triangle QRS$  where Q (-1, -3), R (3, -3) and S (1, 6), then find :

**[a]** The length of  $\overline{QR}$

**[b]** The type of the triangle QRS according to its side lengths.

**[c]** The number of axes of symmetry for the triangle QRS

5

- 1 On a square lattice , draw  $\triangle ABC$  where  $A(5, 3)$  ,  $B(1, 1)$  and  $C(6, -3)$  , then find its image by translation  
 $(X, y) \longrightarrow (X - 4, y + 1)$

5

- 2 On the coordinate plane , determine the points  $A(1, 2)$  ,  $B(-2, 2)$  and  $C(-2, -4)$  , then find :

5

[a] The length of  $\overline{AB}$ [b] The length of  $\overline{BC}$ [c] The image of  $\triangle ABC$  by translation  $(3, -1)$ 

- 3 If  $A(1, -1)$  and  $B(-1, 3)$  , write the mapping rule of the translation that makes  $B$  the image of  $A$

5

- 4 On a square lattice , draw  $\triangle MNT$  where  $M(1, -3)$  ,  $N(-3, 1)$  and  $T(-2, -5)$  , then draw its image by translation of magnitude 3 units in the positive direction of  $y$ -axis.

5

- 5 Choose the correct answer :

5

[a] If  $A(-2, 1)$  and  $B(3, 1)$  , then the length of  $\overline{AB} = \dots\dots\dots$  units.

( 0 or 1 or 3 or 5 )

[b] The image of the point  $(2, 3)$  by translation  $(X + 1, y + 2)$  is  $\dots\dots\dots$ 

( (3, 4) or (3, 5) or (4, 3) or (5, 3) )

[c] The image of the point  $(-1, 2)$  by translation of magnitude of 3 units in the positive direction of  $y$ -axis is  $\dots\dots\dots$ 

( (-1, 3) or (2, 2) or (-1, 5) or (2, 5) )

[d] The image of the point  $M(3, 4)$  by translation  $\dots\dots\dots$  is  $(4, 3)$ 

( (1, 1) or (-1, -1) or (-1, 1) or (1, -1) )

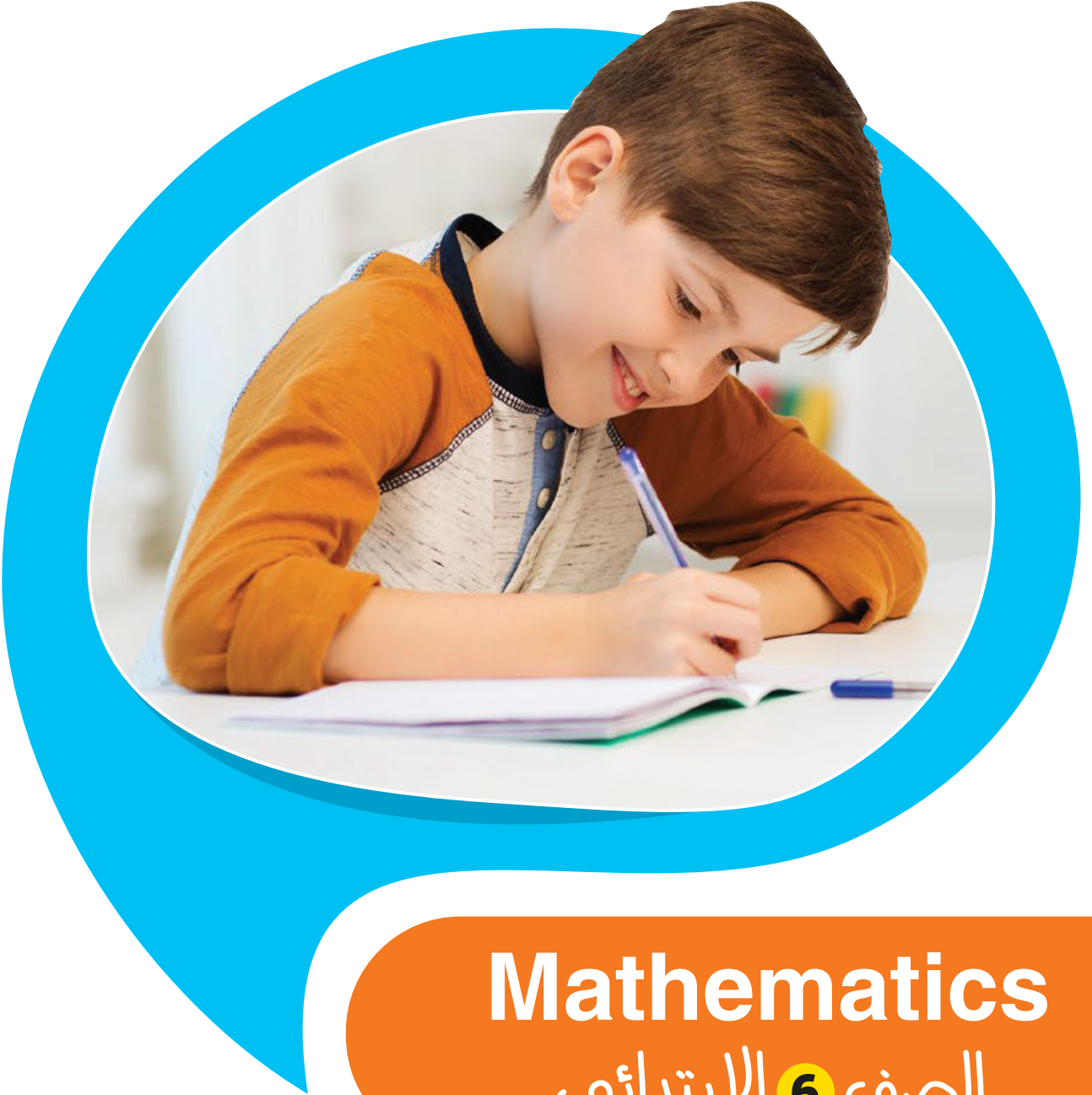
[e] If  $X(-3, 2)$  ,  $Y(-3, 4)$  , then the length of  $\overline{XY} = \dots\dots\dots$  units.

( 2 or 3 or 4 or 5 )



**ALADWANA**

**Gem**



**Mathematics**

الصف 6 الابتدائي

مقترح النماذج الاسترشادية لشهر مارس

العام الدراسي 2022 - 2023

## 1 Choose the correct answer:

3

a If  $-|-3| = x$ , then  $x =$  .....

• 3

• -3

•  $\pm 3$ 

• 1

b  $Z^+ \cap Z^- =$  .....

• zero

•  $\emptyset$ •  $Z$ •  $Z^+$ c The coordinates of the point A is ....., if its image by the translation  $(1, -5)$  is  $A'(2, 0)$ .•  $(3, -5)$ •  $(1, -5)$ •  $(1, 5)$ •  $(3, 5)$ 

## 2 Answer the following:

2

a Use the properties of addition to find:

$$8 + 20 + (-8)$$

.....

.....

.....

.....

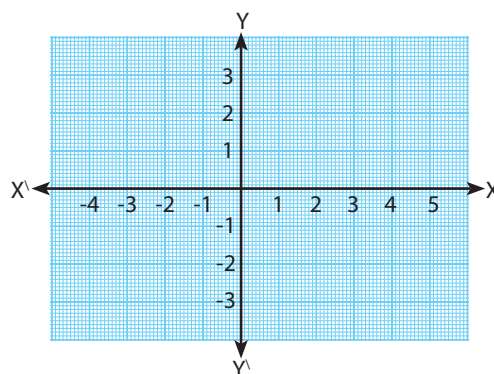
b On the coordinate plane, determine the points A (5, 1), B (5, -3), C (-3, -3) and

D (-3, 1), then find:

The length of  $\overline{AB}$  and  $\overline{AD}$ .

.....

.....





1 Choose the correct answer:

a  $\mathbb{Z}^+ \dots \mathbb{N}$ ☐  $\in$ ☐  $\notin$ ☐  $\subset$ ☐  $\not\subset$ b If  $x \in \{2, -3\} \cap \{5, -3\}$ , then  $x = \dots$ ☐ 2☐ -3☐ -5☐ 5c  $-|-8| \dots -12$ ☐  $>$ ☐  $<$ ☐  $=$ ☐ otherwise

2 In the opposite figure: ABC is a triangle in which

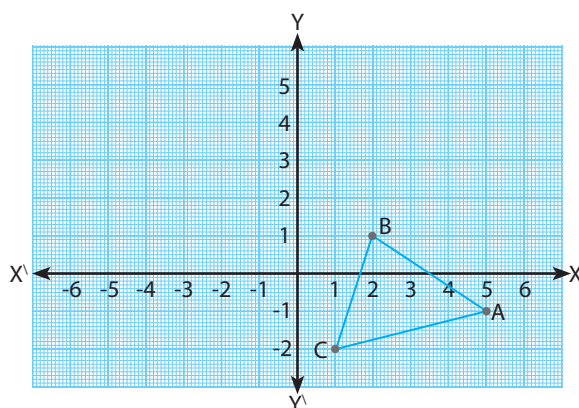
A (5, -1), B (2, 1) and C (1, -2). Find the image of the triangle ABC

by translation  $(x - 4, y + 2)$ . Represent the image of the triangle ABC on the coordinates plane:

.....

.....

.....



## 1 Choose the correct answer:

a The additive inverse of (3) is .....

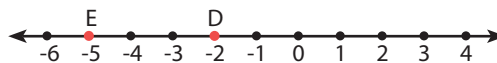
☐  $\frac{1}{3}$ ☐ 3☐ -3☐ 0

b The greatest non-positive integer is .....

☐ 1☐ -1☐ 0☐ millionc  $\left| \frac{5-11}{3} \right| \dots\dots\dots \mathbb{Z}^+$ ☐  $\in$ ☐  $\notin$ ☐  $\subset$ ☐  $\not\subset$ 

## 2 Calculate the distance between the two points D and E

on the following number line:



---

---

## Model (1)

## 1 Choose the correct answer:

3

a If  $-|-3| = x$ , then  $x =$  .....

• 3

• -3

•  $\pm 3$ 

• 1

b  $Z^+ \cap Z^- =$  .....

• zero

•  $\emptyset$ 

• Z

•  $Z^+$ c The coordinates of the point A is ....., if its image by the translation  $(1, -5)$  is  $A'(2, 0)$ .•  $(3, -5)$ •  $(1, -5)$ •  $(1, 5)$ •  $(3, 5)$ 

## 2 Answer the following:

2

a Use the properties of addition to find:

$$8 + 20 + (-8)$$

$$= 8 + (-8) + 20 \quad (\text{commutative property})$$

$$= [8 + (-8)] + 20 \quad (\text{associative property})$$

$$= 0 + 20 \quad (\text{additive inverse property})$$

$$= 20 \quad (\text{additive identity property})$$

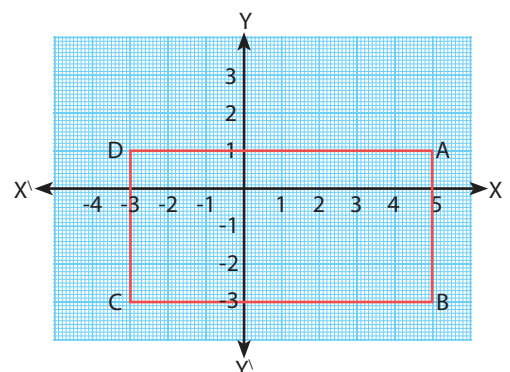
b On the coordinate plane, determine the points A (5, 1), B (5, -3), C (-3, -3) and

D (-3, 1), then find:

The length of  $\overline{AB}$  and  $\overline{AD}$ .

$$\text{Length of } \overline{AB} = 4 \text{ length units}$$

$$\text{Length of } \overline{AD} = 8 \text{ length units}$$



1 Choose the correct answer:

a  $\mathbb{Z}^+ \dots \mathbb{N}$ ☐  $\in$ ☐  $\notin$ ☐  $\subset$ ☐  $\not\subset$ b If  $x \in \{2, -3\} \cap \{5, -3\}$ , then  $x = \dots$ ☐ 2☒ -3☐ -5☐ 5c  $-|-8| \dots -12$ ☐  $>$ ☐  $<$ ☐  $=$ ☐ otherwise

2 In the opposite figure: ABC is a triangle in which

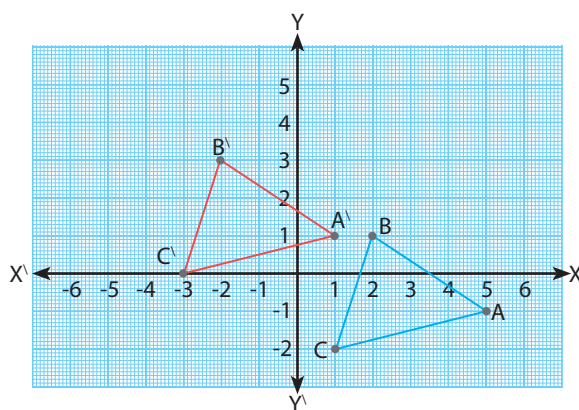
A (5, -1), B (2, 1) and C (1, -2). Find the image of the triangle ABC

by translation  $(x - 4, y + 2)$ . Represent the image of the triangle ABC on the coordinates plane:

$$A(5, -1) \xrightarrow{(-4, 2)} A'(1, 1)$$

$$B(2, 1) \xrightarrow{(-4, 2)} B'(-2, 3)$$

$$C(1, -2) \xrightarrow{(-4, 2)} C'(-3, 0)$$





## 1 Choose the correct answer:

a The additive inverse of (3) is .....

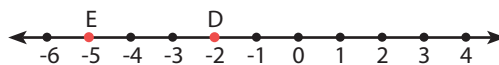
☐  $\frac{1}{3}$ ☐ 3☒ -3☐ 0

b The greatest non-positive integer is .....

☐ 1☐ -1☒ 0☐ millionc  $\left| \frac{5-11}{3} \right|$  .....  $\mathbb{Z}^+$ ☒  $\in$ ☐  $\notin$ ☐  $\subset$ ☐  $\not\subset$ 

## 2 Calculate the distance between the two points D and E

on the following number line:



$$DE = |-2 - (-5)| = 3 \text{ units}$$

## Worksheet 1 On Lesson (1) – Unit (1)

Total mark

25

### 1 Choose the correct answer:

a)  $|\frac{1}{3}| - |-\frac{1}{3}| \dots\dots\dots \mathbb{Z}$

( $\in$  or  $\notin$  or  $\subset$  or  $\emptyset$ )

b) If  $x = |-5|$ , then  $x = \dots\dots\dots$

(-5 or 5 or 0 or 10)

c)  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

( $\mathbb{Z}$  or  $\mathbb{Z}^+$  or  $\mathbb{Z}^-$  or  $\emptyset$ )

d)  $\mathbb{N} \cup \mathbb{Z} = \dots\dots\dots$

( $\mathbb{Z}$  or  $\mathbb{N}$  or  $\mathbb{Z}^-$  or  $\mathbb{Z}^+$ )

e)  $\mathbb{Z}^+ - \mathbb{Z}^- = \dots\dots\dots$

( $\mathbb{Z}^+$  or  $\emptyset$  or  $\mathbb{N}$  or  $\{0\}$ )

5

5

5

5

5

### 2 Complete each of the following:

a) The complement of  $\mathbb{Z}^-$  with respect to  $\mathbb{Z}$  is  $\dots\dots\dots$

b)  $|-3| + |-2| = \dots\dots\dots$

c)  $\{15\} \dots\dots\dots \mathbb{Z}^-$

d) If  $X \subset \{2, -3\} \cap \{5, -3\}$ , then  $X = \dots\dots\dots$  or  $\dots\dots\dots$

e)  $\frac{|-6| + 2}{2} = \dots\dots\dots$

### 3 Represent each of the following on the number line:

a)  $-3, 0, 2, 1, -6, 5$

b)  $6, -3, 0, -1, 3, 5$

### 4 Find the result of each of the following:

a)  $|-4| \times |7|$

b)  $|-30| \div |-5|$

### 5 Find the value of $x$ to get a true statement:

a)  $-5 \in \{-1, 0, -3, x\}$

b)  $|-7| \notin \{x, -7, 3\}$

Worksheet **2** On Lesson (1) – Unit (1)

Total mark

25

**1 Choose the correct answer:**

5

a)  $\mathbb{Z}^+ \cup \{0\} = \dots\dots\dots$  ( $\emptyset$  or  $\mathbb{Z}$  or  $\mathbb{N}$  or  $\mathbb{Z}^-$ )

b)  $\frac{7-7}{9} \dots\dots\dots \mathbb{Z}$  ( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

c)  $-2 \dots\dots\dots -|4|$  ( $>$  or  $<$  or  $=$  or otherwise)

d) If  $x = |-6|$ , then  $x = \dots\dots\dots$  (6 or  $-6$  or 3 or zero)

e)  $\emptyset \dots\dots\dots \mathbb{Z}$  ( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

**2 Complete each of the following:**

5

a)  $\mathbb{Z} = \mathbb{Z}^- \cup \dots\dots\dots \cup \dots\dots\dots$

b)  $|-12| = \dots\dots\dots$

c)  $|-5| - 2 = \dots\dots\dots$

d)  $\mathbb{Z} - \mathbb{Z}^- = \dots\dots\dots$

e) If  $|a| = 4$ , then  $a = \dots\dots\dots$  or  $\dots\dots\dots$

**3 Find the result of each of the following:**

5

a)  $|-45| \div |-9|$

b)  $|-7| - |6|$

**4 Put a suitable sign ( $\in$ ,  $\notin$ ,  $\subset$  or  $\not\subset$ ):**

5

a) Zero  $\dots\dots\dots \mathbb{N}$

b)  $-\frac{5}{4} \dots\dots\dots \mathbb{Z}^-$

c)  $\{3, -7\} \dots\dots\dots \mathbb{Z}^-$

d)  $\mathbb{Z}^+ \dots\dots\dots \mathbb{N}$

**5 Find the value of  $x$  to get a true statement:**

5

a)  $-12 \in \{4, -4, x, -1\}$

b)  $-7 \in \{x, -4, -5\} - \{-4, -5\}$

c)  $\{5, x\} \cup \{7, 5\} = \{4, 5, 7\}$

d)  $|-6| \in \{x, -6\}$

Worksheet **3** On Lesson (2) – Unit (1)

Total mark

25

**1 Choose the correct answer:**

5

- a) The integer number which lies between  $-1$  and  $4$  is ..... ( $-3$  **or**  $-2$  **or**  $1$  **or**  $5$ )
- b) The integer number just before  $-6$  is ..... ( $-4$  **or**  $-5$  **or**  $-7$  **or** zero)
- c)  $-|8|$  .....  $-|-8|$  ( $>$  **or**  $<$  **or**  $=$  **or** otherwise)
- d)  $-1 >$  ..... ( $0$  **or**  $1$  **or**  $2$  **or**  $-2$ )
- e) The smallest counting number is ..... ( $0$  **or**  $1$  **or**  $-1$  **or**  $2$ )

**2 Complete each of the following:**

5

- a) The previous integer and the next integer of  $-3$  are ..... and .....
- b) The set of integers between  $-3$  and  $2$  is .....
- c) The number of integers between  $-5$  and  $5$  is .....
- d) The greatest negative integer is .....
- e) The number ..... is neither positive nor negative.

**3 Write each of the following sets using the listing method:**

5

- a) The set of integers which are greater than  $-7$
- b)  $X = \{x : x \in \mathbb{Z}, -4 < x \leq 5\}$

**4 Arrange the following in ascending order:**

5

- a)  $-8, -9, 7, |-13|, |-5|$                       b)  $|-6|, -4, |-7|, \text{zero}, 8$

**5 Which of the following sets of integers is arranged in descending order?**

5

- $|-7|, 6, |-2|, -1$  **or**  $-1, -8, -9, 0$



## Worksheet 4 Till Lesson (2) – Unit (1)

Total mark

25

## 1 Choose the correct answer:

5

- a)  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$  ( $\mathbb{Z}^-$  or  $\{0\}$  or  $\mathbb{Z}^+$  or 0)
- b) The smallest positive integer is  $\dots\dots\dots$  (1 or -1 or  $\emptyset$  or zero)
- c) The number of integers between -2 and 4 is  $\dots\dots\dots$  (3 or 4 or 5 or 6)
- d)  $|-9| + 3 \dots\dots\dots \mathbb{Z}$  ( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )
- e) If  $a \in \{2, -5, -3\} \cap \{-2, 5, -3\}$ , then  $a = \dots\dots\dots$  (-5 or -3 or 20 or 3)

## 2 Complete each of the following:

5

- a) The previous integer and the next integer of -40 are  $\dots\dots\dots$  and  $\dots\dots\dots$
- b) If  $x = |-7|$ , then  $x = \dots\dots\dots$
- c) If  $|x| = 15$ , then  $x = \dots\dots\dots$  or  $\dots\dots\dots$
- d)  $3 + |-3| \dots\dots\dots |-10|$  (Put  $<$  or  $>$  or  $=$ )
- e)  $|-3| - |-5| + 2 = \dots\dots\dots$

## 3 Arrange each of the following in descending order:

5

- a)  $|-2|$ , 3, 4, -2 and zero
- b) -3, zero,  $|-2|$ , 5 and  $|-7|$

## 4 Represent the following sets on the number line:

5

- a) The set of integers less than 4
- b) The set of integers greater than -3 and less than 4

5 Find the value of  $x$  in each of the following:

5

- a)  $|-2| + |3| = x$
- b)  $|x| = 3 + |-5|$

**Total mark**

25

## 5

- a)  $12 - |-3| = \dots\dots\dots$  (15 or 9 or  $-9$  or  $-15$ )
- b)  $-|5 - 6| = \dots\dots\dots$  (1 or  $-1$  or 0 or  $-11$ )
- c) The additive identity in  $\mathbb{Z}$  is  $\dots\dots\dots$  (0 or 1 or  $-1$  or 2)
- d) Zero  $- (-3) = \dots\dots\dots$  ( $-3$  or 3 or Zero or  $-6$ )
- e) If  $x = -2$ , then  $x + (-2) = \dots\dots\dots$  (4 or 0 or  $-2$  or  $-4$ )

## 5

- a) The result of subtracting  $(-6)$  from  $-16$  is .....
- b) The additive inverse of  $(-8)$  is .....
- c) If  $a = -12$ ,  $b = -4$ , then  $a - b =$  .....
- d)  $(4 + 5) + (-3) =$  ..... +  $[5 + (-3)]$
- e)  $-8 +$  ..... =  $-13$

## 5

- a)  $140 + 30 - 140$                       b)  $135 - 100 + 65 + 1100$

## 5

- a)  $a + b + c$                       b)  $a - b + c$                       c)  $(-b + a) - c$

## 5

- a)  $\mathbb{Z}$  is closed under addition operation. ( )
- b) The additive identity in  $\mathbb{Z}$  is 1. ( )
- c) The operation of subtraction is commutative in  $\mathbb{Z}$ . ( )
- d) For every  $a \in \mathbb{Z}$ , then  $(-a) \in \mathbb{Z}$ , where  $a + (-a) = \text{zero}$  ( )

## Worksheet 6 Till Lesson (3) – Unit (1)

Total mark

25

## 1 Choose the correct answer:

5

a)  $\left| \frac{7-11}{2} \right| = \dots\dots\dots$

(1 or 0 or 2 or -2)

b)  $-4 - (-6) \dots\dots\dots 0$

(> or < or = or  $\leq$ )

c)  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

( $\emptyset$  or  $\{0\}$  or  $\mathbb{Z}$  or  $\mathbb{Z} - \{0\}$ )

d)  $\mathbb{Z} - \mathbb{Z}^- = \dots\dots\dots$

( $\mathbb{Z}^-$  or  $\mathbb{Z}^+$  or  $\mathbb{N}$  or  $\{0\}$ )

e)  $|-5| + \dots\dots\dots = \text{zero}$

(-5 or 5 or Zero or 1)

## 2 Complete each of the following:

5

a)  $8 + (-11) = \dots\dots\dots$

b)  $(-7) + (-4) = \dots\dots\dots$

c) The greatest negative integer =  $\dots\dots\dots$

d)  $-|7| + |-5| = \dots\dots\dots$

e)  $23 - (-13) = \dots\dots\dots$

3 a) Use the properties of addition in  $\mathbb{Z}$  to get the result of the following:

5

(1)  $(-240) + 34 + 140$

(2)  $1\,035 + 75 + (-1\,000)$

b) Check the closure property in addition and subtraction on the set  $X = \{-2, 0, 2, 3\}$ .

## 4 a) Complete each of the following:

5

(1) The additive inverse of 7 is  $\dots\dots\dots$

(2) The additive identity element in  $\mathbb{Z}$  is  $\dots\dots\dots$

b) If  $a = 4$ ,  $b = -2$  and  $c = -5$ , find:

(1)  $a - (b - c)$

(2)  $(b + c) - a$

## 5 Use the number line to find the sum of each of the following:

5

a)  $3 + 2$

b)  $(-3) + 4$

c)  $3 + (-7)$



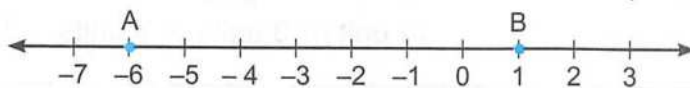
## Worksheet 19 On Lesson (1) – Unit (3)

Total mark

25

### 1 Choose the correct answer:

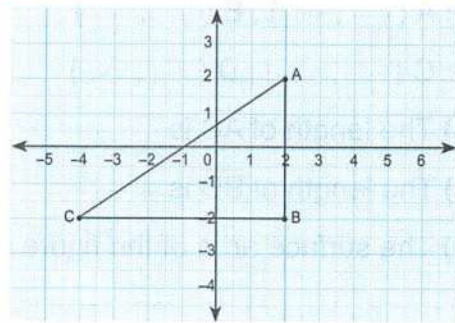
- a) If A (2, 7), B (2, 3), then the length of  $\overline{AB}$  = ..... units. (3 or 4 or 5 or 7)
- b) If B (4, 3), C (4, 7), then the length of  $\overline{BC}$  = ..... units. (5 or 2 or 4 or -4)
- c) If X (2, 3), Y (6, 3), then the length of  $\overline{XY}$  = ..... units. (3 or 4 or 5 or 6)
- d) In the following figure, the distance between points A and B = ..... units.



(5 or 6 or 7 or 8)

### 2 On the opposite coordinate plane: Complete each of the following:

- a) A (....., .....), B (....., .....), C (....., .....)
- b) The length of  $\overline{BC}$  = .....
- c) The length of  $\overline{BA}$  = .....
- d) The type of the triangle ABC with respect to its angles is .....



- 3 In the coordinate plane, locate the points A (0, 1), B (4, 3), C (4, 7), then **find** the length of  $\overline{BC}$ .

- 4 On the coordinate plane, determine the points A (5, 1), B (5, -3), C (-3, -3) and D (-3, 1), then **find**:

- a) The length of  $\overline{AB}$  and  $\overline{AD}$ .
- b) The surface area and the perimeter of ABCD.

### 5 Write the missing coordinate to make each statement true:

- a) If A (-3, 5) and B (x, 5) where  $AB = 5$  units, then x = ..... or .....
- b) If C (-6, -3) and D (-6, y) where  $CD = 6$  units, then y = ..... or .....
- c) If E (-2, -2) and F (-2, z) where  $EF = 9$  units, then z = ..... or .....
- d) If L (1, 1) and M (m, 1) where  $LM = 4$  units, then m = ..... or .....

Worksheet **20** On Lesson (1) – Unit (3)

Total mark

25

**1** Choose the correct answer:

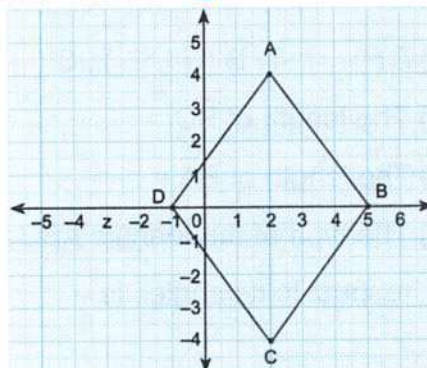
5

- a) If A (−3, 2) and B (−3, −4), then the length of  $\overline{AB}$  = ..... units. (5 or 6 or 7 or 8)
- b) Having A (−2, 1) and B (3, 1), then the length of  $\overline{AB}$  = ..... units. (4 or 5 or 6 or 7)
- c) If M (−4, 4) and N (1, 4), then the length of  $\overline{MN}$  = ..... units. (−5 or 5 or 3 or −3)
- d) Which of the following values could be the Y. Coordinate of the point (10, ..... ) that is 13 units from (10, 6)?  
(1 unit or 3 units or 7 units or 9 units)

**2** In the opposite figure: Complete each of the following:

5

- a) A (....., .....), B (....., .....),  
C (....., .....), D (....., .....)
- b) The length of  $\overline{AC}$  is .....
- c) The length of  $\overline{BD}$  is .....
- d) The surface area of the figure ABCD = .....



- 3** On the coordinate plane, **determine** the points A (2, 1), B (2, 3), C (−3, 3), D (−3, 1), **then write** the name of the shape ABCD?

5

- 4** Determine the following points A (−3, 4), B (1, 4) and C (1, 2) on the coordinate plane, then **find** the length of  $\overline{AB}$  and  $\overline{BC}$ .

5

**5** Find the distance between each two points:

5

- a) (−2, −6) and (−2, 5)
- b) (6, 4) and (6, −8)
- c) (8, −2) and (11, −2)
- d) (8, −10) and (−5, −10)



Worksheet **21** On Lesson (2) – Unit (3)

Total mark

25

**1** Choose the correct answer:

5

- a) The number of axes of symmetry of a square = ..... (1 or 2 or 3 or 4)
- b) The image of the point A (5 , 3) by translation (− 4 , 1) is .....  
 ((1 , 4) or (−1 , − 4) or (3 , 2) or (5 , −7))
- c) The image of the point (− 3 , − 3) by translation 3 units in the positive direction of y-axis is .....  
 ((0 , 0) or (0 , − 3) or (− 3 , 0) or (− 3 , 3))
- d) The image of (3 , 3) by translation ( $x + 2$  ,  $y - 1$ ) is .....  
 ((− 2 , 5) or (2 , 5) or (− 5 , 2) or (5 , 2))

**2** Complete each of the following:

5

- a) If A (2 , 0), then  $\hat{A}$  (..... , ..... ) is the image by translation (2 , 3).
- b)  $\hat{A}$  (3 , 5) is the image of A by translation (2 , 1), then the coordinate of A (..... , .....).
- c) If A (1 , −1) and B (−1 , 3), then the translation that makes B the image of A is .....
- d) The image of the point (− 4 , − 4) by translation ( $x - 3$  ,  $y + 3$ ) is .....

- 3** On a lattice, **plot** the vertices of the triangle ABC where A (1 , 2), B (− 2 , 2) and C (− 2 , − 4), then **draw** its image by translation ( $x + 3$  ,  $y - 1$ ).

5

- 4** On the coordinate plane, determine A (2 , 3), B (4 , − 2), C (− 2 , − 2), D (− 4 , 3), and E (− 4 , − 2), then **find**:

5

- a) The length of  $\overline{AD}$ ,  $\overline{DE}$  and  $\overline{BC}$  .
- b) The surface area of the shape ABCD and the surface area of  $\Delta DEC$ .
- c) The image of ABCD by translation (2 , − 2).

- 5** **Draw**  $\Delta ABC$ , where A (1 , 1), B (− 3 , − 1) and C (0 , − 5), **then determine** graphically its image by translation (5 , 0).

5

31

Worksheet **22** Till Lesson (2) – Unit (3)

Total mark

25

5

**1 Choose the correct answer:**

- a) The image of the point A  $(-4, 3)$  by translation  $(-1, -4)$  is .....  
 $(-5, -7)$  **or**  $(5, -1)$  **or**  $(-7, 3)$  **or**  $(-5, -1)$
- b) If A  $(3, 5)$  and B  $(3, -2)$ , then the length of AB = ..... units. (6 **or** 7 **or** 8 **or** 9)
- c) The number of axes of symmetry of isosceles triangle is ..... (1 **or** 2 **or** 3 **or** 4)
- d) The image of the point  $(-1, 4)$  by translation  $(1, -3)$  is the point .....  
 $(0, 1)$  **or**  $(0, 0)$  **or**  $(1, 1)$  **or**  $(2, 2)$

**2 Complete each of the following:**

- a) If A  $(1, 2)$ , then the image of A by translation  $(X + 1, Y - 1)$  is .....
- b) The image of the point  $(0, 2)$  by translation  $(X + 1, Y + 3)$  is .....
- c) The image of the point  $(3, -2)$  by translation  $(4, 2)$  is .....
- d) The image of  $\overline{AB}$  where A  $(2, 3)$  and B  $(-2, 0)$  by translation  $(X + 3, Y - 2)$  is .....

**3 Determine the following points on a coordinate plane:**

A  $(1, 5)$ , B  $(1, 2)$  and C  $(3, 2)$ , then determine the image of ABC by translation  $(X + 3, Y)$ .

**4 Determine the following points on a coordinate plane:**

A  $(2, 2)$ , B  $(2, -2)$ , C  $(-2, -2)$  and D  $(-2, 2)$  and find its image by translation  $(X + 2, Y + 1)$ , then answer the following:

- a) Mention the name of the shape.
- b) Calculate the area of the shape.

**5 Determine the image of the shape ABCD on the Cartesian coordinate**

where A  $(2, 3)$ , B  $(2, 1)$ , C  $(-2, 1)$  and D  $(-2, 3)$  by translation  $(X + 3, Y + 3)$ ,

**what** is the type of the shape A'B'C'D'?

# Exercises 1

Put the suitable sign " $\in$ ,  $\notin$ ,  $\subset$  or  $\not\subset$ ":

$$-3 \square \mathbb{N}$$

$$\{-5\} \square \mathbb{Z}$$

$$\text{Zero} \square \mathbb{Z}^+$$

$$\{1, -2\} \square \mathbb{N}$$

$$\mathbb{Z}^- \square \mathbb{Z}$$

$$\{-3, -\frac{1}{3}\} \square \mathbb{Z}$$

$$\frac{13}{5} \square \mathbb{Z}$$

$$|-65| \square \mathbb{Z}^-$$

$$\frac{11-5}{3} \square \mathbb{Z}$$

$$\{2, 5, \frac{3}{7}\} \square \mathbb{Z}$$

$$\frac{7}{12-6} \square \mathbb{Z}$$

$$\frac{11-5}{3} \square \mathbb{Z}$$

**Write an integer to express each situation of the following :**

- 1) Hany gained LE 76 from his saving account. (.....)
- 2) Hany withdrew LE 76 from his saving account. (.....)
- 3) The temperature of Moscow City is 8 degrees below Zero. (.....)
- 4) The temperature of Moscow City is 10 degrees above Zero. (.....)
- 5) Building a public garage consists four floors underground. (.....)
- 6) Building a public garage consists 2 floors underground. (.....)
- 7) Paris rises 6 metres above sea level. (.....)
- 8) A Submarine at a depth of 90 metres below sea level (.....)
- 9) Ahmed withdrew 6000 pounds from his bank account. (.....)
- 10) The school added 10 marks for the student (Sarah), for her excellence in artistic activity. (.....)

**Complete the following**

$$(a) |-12| = \dots$$

$$(b) -|-105| = \dots$$

$$(c) |-5| + |5| = \dots$$

$$(d) -|-10| = \dots$$

$$(e) |5| + |-7| = \dots$$

$$(f) \text{ the relation between } |b|, |-b| \text{ is } \dots$$

$$(g) |-5| + |5| = \dots$$

$$(h) -|-251| = \dots$$

$$(c) |-5| \times |5| = \dots$$

$$(d) -|2| = \dots$$



**Express each of the following sets using the listing method:**

(a) The set of integers which are less than 3

$$A = \{ \dots \}$$

(b) The set of integers which are less than 0

$$B = \{ \dots \}$$

(c) The set of integers which are less than -5

$$C = \{ \dots \}$$

(d) The set of integers which are less than -1

$$D = \{ \dots \}$$

(e) The set of integers which are greater than -5.

$$E = \{ \dots \}$$

(f) The set of integers between -4 and 4

$$F = \{ \dots \}$$

(g) The set of integers between -8 and 1

$$G = \{ \dots \}$$

(h) The set of integers which are greater than -2.

$$H = \{ \dots \}$$

(i) The set of integers which are less than 6 and greater than -2.

$$I = \{ \dots \}$$

(j) The set of integers which are less than 3 and greater than -3

$$J = \{ \dots \}$$

(k) The set of non - positive even integers.

$$K = \{ \dots \}$$

**Write the inverse of each of the numbers :**

113  $\rightarrow$  ( ..... )      -45  $\rightarrow$  ( ..... )      -9  $\rightarrow$  ( ..... )      13  $\rightarrow$  ( ..... )

0  $\rightarrow$  ( ..... )      -99  $\rightarrow$  ( ..... )      7  $\rightarrow$  ( ..... )      1  $\rightarrow$  ( ..... )

**Complete the following using one of the words (positive - negative - Zero) :**

- (a) Moving forward is represented by ..... numbers, while, moving backward is represented by ..... numbers.
- (b) Moving to the right is represented by ..... numbers, while moving to the left is represented by ..... numbers.
- (c) Lowering than sea level is represented by ..... numbers, Height above sea level is represented by ..... numbers. Sea level is represented by the number .....

### Complete :

(1)  $|-5| = \dots\dots\dots$

(13)  $\mathbb{Z}^- \dots\dots\dots \mathbb{N}$

(2) If  $|x| = 5$ , then  $x = \dots\dots\dots$  or  $\dots\dots\dots$  (14)  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$

(3)  $3 + |-3| = \dots\dots\dots$

(15)  $\mathbb{Z} - \mathbb{Z}^- = \dots\dots\dots$

(4)  $\{15\} \dots\dots\dots \mathbb{Z}^-$

(16)  $\mathbb{Z}^+ \cup \{0\} = \dots\dots\dots$

(5)  $\mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^- = \dots\dots\dots$

(17)  $\mathbb{Z}^+ \cup \dots\dots\dots = \mathbb{N}$

(6)  $\mathbb{Z} = \mathbb{N} \cup \dots\dots\dots$

(18)  $\mathbb{Z}^+ \cup \mathbb{N} = \dots\dots\dots$

(7)  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

(19)  $\mathbb{N} - \mathbb{Z}^+ = \dots\dots\dots$

(8)  $\mathbb{Z} \cap \mathbb{N} = \dots\dots\dots$

(20)  $\mathbb{N} - \mathbb{Z} = \dots\dots\dots$

(9)  $\mathbb{Z} = \mathbb{Z}^+ \cup \mathbb{Z}^- \cup \dots\dots\dots$

(10) The set of odd integers  $\cup$  the set of even integers =  $\dots\dots\dots$

(11) The complement of  $\mathbb{Z}^-$  with respect to  $\mathbb{Z} = \dots\dots\dots$

(12) The complement of  $\mathbb{Z}^+$  with respect to  $\mathbb{N} = \dots\dots\dots$

### Choose the correct answer :

(1)  $\{2\} \dots\dots\dots \mathbb{Z}$

( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

(2)  $\frac{13}{5} \dots\dots\dots \mathbb{Z}$

( $\notin$  or  $\in$  or  $\subset$  or  $\not\subset$ )

(3)  $\frac{6-6}{8} \dots\dots\dots \mathbb{Z}$

( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

(4)  $|-9| + 3 \dots\dots\dots \mathbb{Z}$

( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

(5)  $|-5| + |7| = \dots\dots\dots$

(12 or 2 or -2 or -12)

(6)  $|-3| + |-2| = \dots\dots\dots$

(-5 or 5 or -1 or 1)



- (7)  $3 + |-3| = \dots\dots\dots$  (0 or 3 or -6 or 6)
- (8) If  $b = |-7|$ , then  $b = \dots\dots\dots$  (-7 or 7 or 0 or 14)
- (9)  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$  ( $\mathbb{Z}$  or  $\mathbb{Z}^+$  or  $\mathbb{Z}^-$  or  $\emptyset$ )
- (10)  $\mathbb{Z} - \mathbb{Z}^- = \dots\dots\dots$  ( $\mathbb{Z}^-$  or  $\mathbb{Z}^+$  or  $\mathbb{N}$  or {zero})
- (11)  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$  ( $\mathbb{N} - \{0\}$  or  $\mathbb{Z}^+$  or  $\mathbb{Z}^-$  or  $\mathbb{Z}$ )
- (12)  $\mathbb{Z}^+ \cup \{0\} = \dots\dots\dots$  ( $\mathbb{Z}$  or  $\mathbb{Z}^-$  or  $\mathbb{N}$  or  $\emptyset$ )
- (13)  $\mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^- = \dots\dots\dots$  ( $\mathbb{N}$  or  $\mathbb{Z}^+$  or  $\emptyset$  or  $\mathbb{Z}$ )
- (14)  $\mathbb{N} \cup \mathbb{Z} = \dots\dots\dots$  ( $\mathbb{Z}$  or  $\mathbb{N} - \{0\}$  or  $\mathbb{Z}^-$  or  $\mathbb{Z}^+$ )
- (15)  $\mathbb{Z} \cap \mathbb{N} = \dots\dots\dots$  ( $\mathbb{Z}$  or  $\mathbb{N}$  or  $\mathbb{Z}^-$  or  $\emptyset$ )
- (16)  $\mathbb{N} \cup \mathbb{Z}^- = \dots\dots\dots$  ( $\mathbb{Z}^-$  or  $\mathbb{Z}^+$  or  $\mathbb{Z}$  or  $\emptyset$ )
- (17)  $\mathbb{Z}^+ - \mathbb{Z}^- = \dots\dots\dots$  ( $\mathbb{Z}^+$  or  $\emptyset$  or  $\mathbb{N}$  or  $\{0\}$ )
- (18) If  $X \subset \{2, -3\} \cap \{5, -3\}$ , then  $X = \dots\dots\dots$  ( $\{2\}$  or  $\{-3\}$  or  $\{-5\}$  or  $\{5\}$ )
- (19)  $|\frac{1}{3}| - |\frac{-1}{3}| \dots\dots\dots \mathbb{Z}$  ( $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$ )

Represent the following sets of numbers on the number line :

- $\{3, -4, 1, -2\}$
- $\{6, -3, 0, -1, 3, 5\}$
- $\{-2, -1, 0, 1, 2\}$
- $\{-3, 0, 2, 1, -6, 5\}$
- $\{-4, -5, -6, \dots\}$



Sheet 1

1 Complete each of the following :

[a]  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

[b]  $|-13| = \dots\dots\dots$

[c] The opposite of 7 is  $\dots\dots\dots$

[d]  $\mathbb{Z} - \mathbb{N} = \dots\dots\dots$

[e]  $\mathbb{N} \cap \mathbb{Z} = \dots\dots\dots$

2 Put the suitable sign " $\in, \notin, \subset$  or  $\not\subset$ " :

[a]  $-7 \square \mathbb{N}$

[b]  $\frac{9}{3} \square \mathbb{Z}$

[c]  $\mathbb{Z}^+ \square \mathbb{N}$

[d]  $|-35| \square \mathbb{N}$

[e]  $\mathbb{N} \square \mathbb{Z}^-$

3 Write an integer to represent each situation.

[a] A temperature of 2 degrees below zero.

( $\dots\dots\dots$ )

[b] An increase of L.E. 7

( $\dots\dots\dots$ )

[c] 9 m. above the sea level.

( $\dots\dots\dots$ )

[d] A loss of P.T. 80

( $\dots\dots\dots$ )

[e] A bank deposit of L.E. 95

( $\dots\dots\dots$ )

4 Find the result of each of the following :

[a]  $3 + |-3| = \dots\dots\dots$

[b]  $|25| + |-5| = \dots\dots\dots$

[c]  $|-11| + 22 = \dots\dots\dots$

[d]  $|-6| \times |3| = \dots\dots\dots$

[e]  $|-9| \times 0 = \dots\dots\dots$

5 Represent the following numbers on the number line :

[a]  $-3$



[b]  $|-5|$



[c]  $-|2|$



[d]  $\{7, 8, 9\}$



[e]  $\{-3, -4, -5, -6, \dots\}$



# Exercises 2

Put [ $<$ ,  $>$  or  $=$ ] :

$3 \square - 3$

$-12 \square - 3$

$|-4| \square |0|$

$-15 \square |-2|$

$3 + |-3| \square 8$

$4 \square 3$

$3 \square -6$

$8 \square |-8|$

$-6 \square -|-3|$

$-3 \square -4$

$|-4| \square 2$

$|-13| \square 3$

$|-4| \square 2$

$|-9| - |-5| \square |-4|$

Complete the following :

- The number ..... is neither positive nor negative.
- The largest negative integer is .....
- ..... is the smallest positive integer.
- The smallest non-negative integer is .....
- The largest non-positive integer is .....
- The set of integers between  $-3$  and  $2 =$  .....
- The set of integers which are less than  $1$  and more than  $-4$  is  
{.....}

Choose the correct answer :

- The smallest positive number is ..... (  $1$  or  $-1$  or  $\emptyset$  or zero )
- $-7$  .....  $|-9|$  (  $>$  or  $=$  or  $<$  or  $\leq$  )
- $(-4)$  .....  $|-4|$  (  $>$  or  $<$  or  $=$  or  $\geq$  )
- An integer included between  $-2$  and  $3$  is ..... (  $-3$  or  $-2$  or  $-1$  or  $3$  )
- The greatest negative integer is ..... (  $0$  or  $-1$  or  $-100$  or  $1$  )
- The integer number just before the number  $-5$  is .....  
(  $-6$  or  $-4$  or  $4$  or  $6$  )
- The integer next to the number  $23$  is ..... (  $25$  or  $22$  or  $23$  or  $24$  )
- The number of integers between  $-2$  and  $2 =$  ... (  $2$  or  $3$  or  $4$  or  $5$  )

Arrange the following once in an ascending order and another in a descending order :

1 , - 5 , - 1 , 3

The ascending order is : , , ,

The descending order is : , , ,

- 22 , 11 , - 11 , 0 , 7

The ascending order is : , , , ,

The descending order is : , , , ,

- 7 , - 9 , - 4 , - 1

The ascending order is : , , ,

The descending order is : , , ,

- 3 , 5 , 2 , - 7 , 10 , - 6

The ascending order is : , , , , ,

The descending order is : , , , , ,

- 3 , 5 , 2 , - 7 , 10 , - 6

The ascending order is : , , , , ,

The descending order is : , , , , ,

8 , - 62 , - 19 , - 42 , 0

The ascending order is : , , , ,

The descending order is : , , , ,

- 6 , 15 , zero ,  $|- 9 |$  , - 18

The ascending order is : , , , ,

The descending order is : , , , ,

- 8 , 12 ,  $|- 8 |$  , - 15 , 19

The ascending order is : , , , ,

The descending order is : , , , ,



1 Put the suitable relation "> , = or <" :

[a]  $-8$    $4$

[b]  $0$    $-2$

[e]  $|-9|$    $-|-10|$

[c]  $5$    $|-5|$

[d]  $-3$    $-5$

2 [a] Arrange the following numbers in an ascending order :

$-6, 15, 0, |-9|$  and  $-18$

..... , ..... , ..... , .....

[b] Arrange the following numbers in a descending order :

$-9, 17, |-9|, -15$  and  $18$

..... , ..... , ..... , ..... , .....

3 Complete each of the following :

[a] The number ..... is neither positive nor negative.

[b]  $-4, -3, -2, \dots, \dots, \dots$  (in the same pattern)

[c]  $\mathbb{Z} - \mathbb{Z}^- = \dots$

[d] The smallest positive integer is .....

[e]  $|-12| + |-21| = \dots$

4 Write

[a] The previous integer and the next integer of  $-27$  ..... , .....

[b] The integers between the two integers  $-5$  and  $3$  .....

5 Write using the listing method each of the following sets :

[a] The set of integers greater than  $-4$  .....

[b] The set of integers smaller than  $-1$  .....

[c] The set of non-negative integers. ....

[d] The set of integers smaller than  $5$  and greater than  $-6$

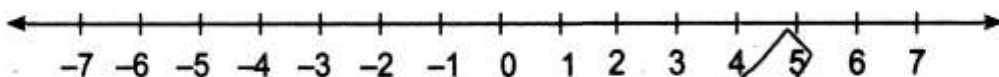
.....



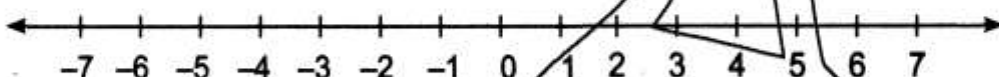
# Exercises 3

· Use the number line to represent the following operations of addition and subtraction :

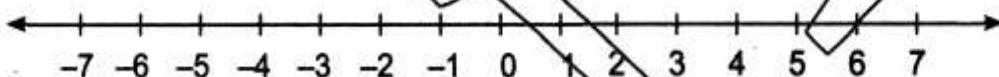
$$2 + 4 = \dots\dots$$



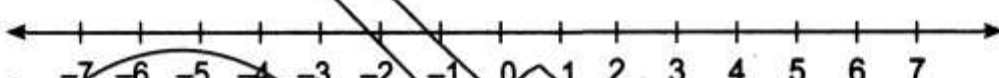
$$3 + 4 = \dots\dots$$



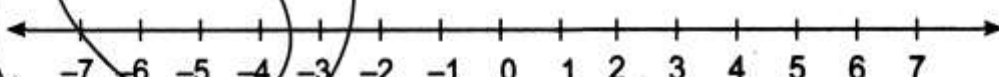
$$(-3) + (-4) = \dots\dots$$



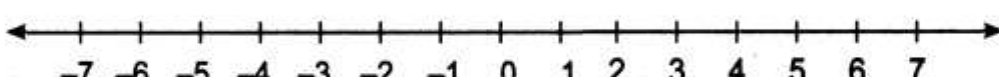
$$(-3) + (-2) = \dots\dots$$



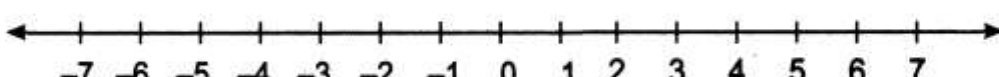
$$3 + (-2) = \dots\dots$$



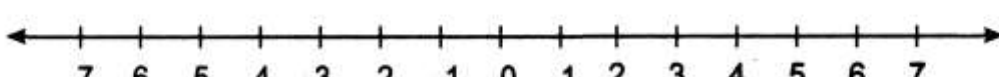
$$6 + (-6) = \dots\dots$$



$$(-3) + 5 = \dots\dots$$

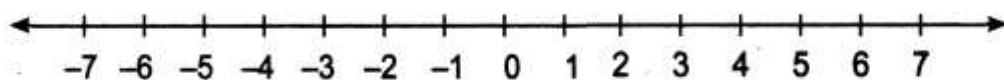


$$(-3) + 2 = \dots\dots$$



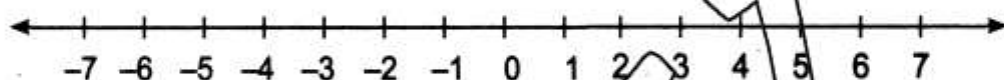
Use the number line to represent the following operations of addition and subtraction :

$$2 - 4$$



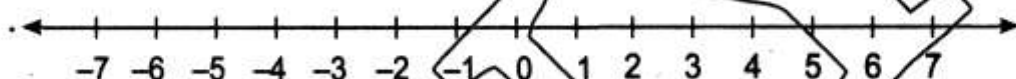
$$= \dots + \dots = \dots$$

$$3 - 4$$



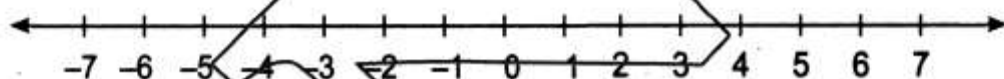
$$= \dots + \dots = \dots$$

$$(-3) - (-4)$$



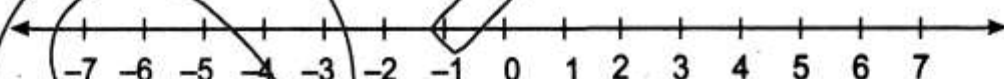
$$= \dots + \dots = \dots$$

$$(-3) - (-2)$$



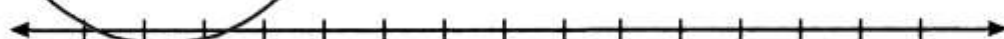
$$= \dots + \dots = \dots$$

$$3 - (-2)$$



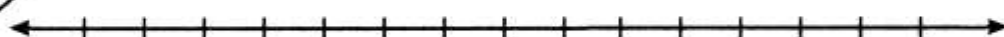
$$= \dots + \dots = \dots$$

$$6 - (-6)$$



$$= \dots + \dots = \dots$$

$$(-3) - 5$$



$$= \dots + \dots = \dots$$

$$(-3) - 2$$



$$= \dots + \dots = \dots$$

**Write the integers representing in each of the following :**

(a)  $x < -1$  .....

(b)  $x > -1$  .....

(c)  $x > 7$  .....

(d)  $x < 7$  .....

(e)  $-4 < x < 4$  .....

(f)  $-2 < x < 3$  .....

(g)  $x < 0$  .....

(h)  $x > 0$  .....

**Find the result of each of the following :**

$-5 + 9 = \dots\dots\dots$  |  $(-2) + (-1) = \dots\dots\dots$  |  $-10 + (-10) = \dots\dots\dots$

$9 + (-8) = \dots\dots\dots$  |  $18 + (-18) = \dots\dots\dots$  |  $0 + (-5) = \dots\dots\dots$

$-6 + 0 = \dots\dots\dots$  |  $-48 + 34 = \dots\dots\dots$  |  $-6 + 9 = \dots\dots\dots$

**Find the result of each of the following :**

$7 - 5 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $3 - 9 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

$0 - 7 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $-7 - 3 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

$19 - (-11) = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $-9 - 8 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

$-3 - (-4) = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $0 - (-3) = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

$(-5) - 0 = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $|-14| - |-28| = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

$-73 - (-73) = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$  |  $33 - |-11| = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

Choose the correct answer :

- (1) The additive identity in  $\mathbb{Z}$  is ..... ( 0 or 1 or -1 or 2 )
- (2)  $-2 + 6 =$  ..... ( 4 or -4 or 8 or -8 )
- (3)  $|\frac{5-8}{3}| =$  ..... ( 1 or 6 or -6 or -2 )
- (4)  $|-5| +$  .....  $= 0$  ( -5 or 5 or 0 or 1 )
- (5) The additive inverse of  $(-5)$  is ..... ( -10 or 5 or 0 or -5 )
- (6)  $4 + (-6) >$  ..... ( 2 or 0 or -2 or -5 )

Write the property of addition in the set  $\mathbb{Z}$  in each of the following :

- (a)  $-5 + 3 = 3 + (-5)$  ( ..... )
- (b)  $6 + (-6) = 0$  ( ..... )
- (c)  $0 + (-7) = -7$  ( ..... )
- (d)  $(-10 + 5) + 3 = -10 + (5 + 3)$  ( ..... )
- (e)  $-a + a = 0$  ( ..... )

Complete each of the following :

- (a)  $4 + (-3) = (-3) +$  ..... | (b)  $5 +$  .....  $= 0$  | (c)  $(-7) +$  .....  $= 0$
- (d)  $(-8) +$  .....  $= (-8)$  | (e)  $6 + (-6) =$  ..... | (f)  $2 - (-3) =$  .....
- (g)  $|-8| - 3 =$  ..... | (h)  $-2 + (\text{.....} + 5) = -2$
- (i)  $(5 + (-8)) + 7 = 5 + (\text{.....} + 7)$  | (j) If  $a + b = b + c$ , then  $c =$  .....
- (k) If  $a + (-3) = b + a$ , then  $b =$  ..... | (l) If  $a + b = b$ , then  $a =$  .....
- (m) The additive identity of integers is ..... | (n) If  $a + b = 0$ , then  $a$  is .....
- (o) The additive inverse of zero is .....
- (p) The additive inverse of number  $(-4)$  is .....

Use the properties of addition operation in Z to find the result :

$$- 5 + (- 6) + 5$$

$$- 7 + 2 + (- 13)$$

$$10 + (- 5) + (- 2)$$

$$(- 17) + 19 + 17$$



Use the properties of addition operation in Z to find the result :

$$25 + (-8) + (-25) + 7$$

.....

.....

.....

.....

.....

$$55 + (-255) + 45 + 255$$

.....

.....

.....

.....

.....

$$2015 + 180 + (-1015)$$

.....

.....

.....

.....

.....

$$63 + 54 + 37 + 46$$

.....

.....

.....

.....

.....

Check the property of closure of the addition and subtraction  
on the following sets of numbers :

$$X = \{-1, 0, 1\}$$

$$Y = \{-2, -1, 0, 1, 2\}$$

$$X = \{-2, 0, 2\}$$

$$Y = \{-3, 0, 1, 2, 3\}$$

If  $a = 3$ ,  $b = -4$  and  $c = -2$ , then find the value of :

$$a + b =$$

$$b + c =$$

$$a - b =$$

$$b - c =$$

$$-c + a - b =$$

$$a + b + c =$$

$$a - b + c =$$

Temperature is recorded in St. Catherine –  $3^{\circ}\text{C}$  at three o'clock after midnight , while it is recorded  $11^{\circ}\text{C}$  in the afternoon.

Calculate the increase in temperature.

The temperature on Sunday morning was  $-2^{\circ}\text{C}$  , the temperature dropped  $7^{\circ}\text{C}$  by Monday and then rose  $5^{\circ}\text{C}$  by Tuesday.

What was the temperature on Tuesday ?

The temperature of the North polar water layer is  $-1^{\circ}\text{C}$ , the temperature rises  $5^{\circ}\text{C}$  in the North Atlantic deep water layer.

What is the temperature of that layer ?

A submarine at a depth of 90 metres below sea level. It rose 60 metres. Use the appropriate calculation to calculate the new depth of the submarine.

Ramy deposited a sum of money amounting to L.E. 6220 , then he withdrew an amount of L.E. 1211 , and then he deposited an another amount of L.E. 2110 How much is the balance of Ramy in the bank ?



Sheet 3

1 Find the result of each of the following :

[a]  $(-7) + 2 = \dots\dots\dots$

[b]  $(-4) + (-5) = \dots\dots\dots$

[c]  $12 + (-12) = \dots\dots\dots$

[d]  $14 - 27 = \dots\dots\dots$

[e]  $16 - (-3) = \dots\dots\dots$

2 Write the property of addition in  $\mathbb{Z}$  in each of the following :

[a]  $(-5 + 6) + 9 = -5 + (6 + 9)$

[b]  $(-8) + 7 = 7 + (-8)$

[c]  $-11 + 0 = -11$

[d]  $14 + (-14) = 0$

3 Complete each of the following :

[a]  $4 + (-7) - 2 = \dots\dots\dots$

[b]  $-8 + 5 + 8 = \dots\dots\dots$

[e] The additive inverse of  $(-5)$  is  $\dots\dots\dots$

[c]  $|-7| + \dots\dots\dots = 0$

[d]  $4 + \dots\dots\dots = -1$

4 Use the properties of addition in  $\mathbb{Z}$  to find :

[a]  $-15 + 29 + 15$

[b]  $55 + (-255) + 45 + 255$

5 Arrange each of the following in an ascending order :

[a]  $|-3|$  ,  $5$  ,  $-3$  ,  $0$  and  $4$

$\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$

[b]  $1$  ,  $11$  ,  $-1$  and  $-11$

$\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$  ,  $\dots\dots\dots$

# Exercises 1

## The distance between two points in the coordinate plane

In the opposite figure :

If the points A , B , C and D represent the numbers  $-6$  ,  $-1$  ,  $0$  and  $3$  respectively , then complete :

AB = .....

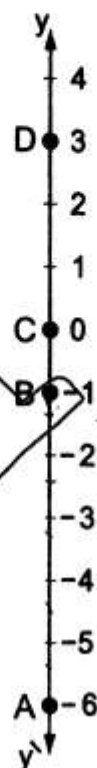
AC = .....

BC = .....

BD = .....

AD = .....

CD = .....



From the following figure complete :



EF = .....

EK = .....

FK = .....

EG = .....

FG = .....

GK = .....

EH = .....

FH = .....

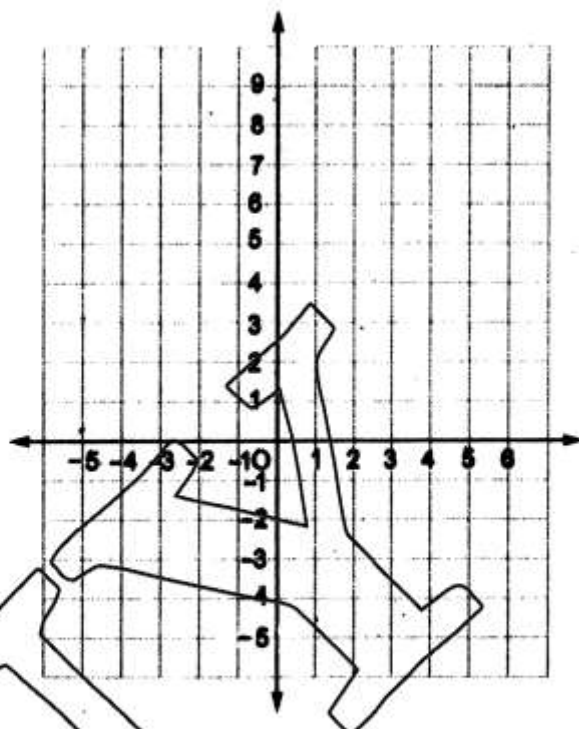
HK = .....



**In the opposite coordinate plane :**

- a** Determine the position of the following points : A ( - 3 , - 3 ) , B ( - 3 , 2 ) , C ( 5 , 2 ) and D ( 5 , - 3 ) and mention the name of the shape ABCD

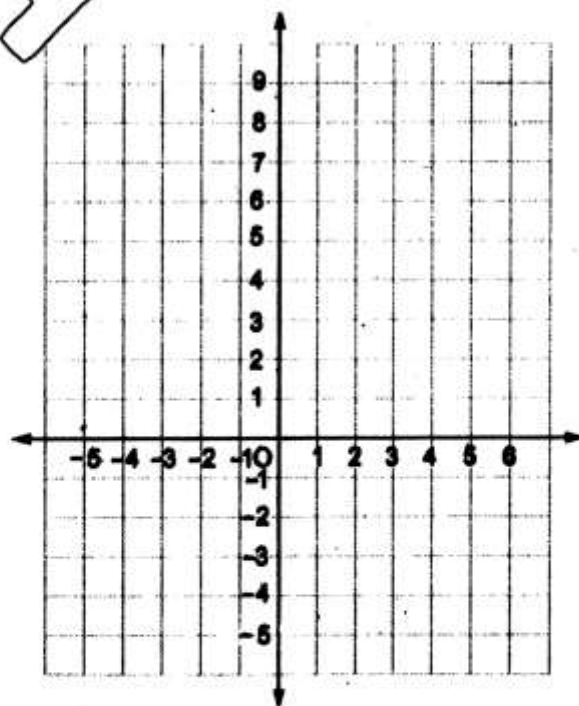
- b** Find the perimeter and the area of the shape ABCD



**In the opposite coordinate plane :**

- a** Determine the position of the following points : L ( - 1 , 1 ) , M ( 1 , 1 ) , N ( 1 , 8 ) and E ( - 1 , 8 )

- b** Find the perimeter and the area of the shape LMNE



- c** Determine whether the shape is symmetric or not ?

Complete each of the following :

- a** The image of the point (2 , 5) by translation

$(x, y) \longrightarrow (x + 2, y + 1)$  is .....

- b** The image of the point (3 , 2) by translation

$(x, y) \longrightarrow (x + 3, y - 2)$  is .....

- c** The image of the point (- 5 , 4) by translation

$(x, y) \longrightarrow (x + 4, y - 5)$  is .....

- d** The image of the point (- 1 , 3) by translation (2 , - 3) is .....

- e** The image of the point (0 , 5) by translation (- 2 , 1) is .....

- f** The image of the point (- 2 , - 5) by translation

$(x, y) \longrightarrow (x - 2, y)$  is .....

- g** The image of the point (3 , - 2) by translation

$(x, y) \longrightarrow (x, y + 3)$  is .....

- h** The image of the point by the translation

$(x, y) \longrightarrow (x - 2, y + 3)$  is (7 , 4)

- i** If the image of the point (3 , 2) is the point (6 , 1) , then the translation

rule is  $(x, y) \longrightarrow (\dots, \dots)$

- j** The image of the point A (3 , 6) by translation 3 units in the negative direction of x-axis is .....

Choose the correct answer :

- a** The image of the point A (1 , 2) by translation (1 , - 1) is .....

[a] (2 , 1)      [b] (2 , 3)      [c] (1 , 1)      [d] (1 , 3)

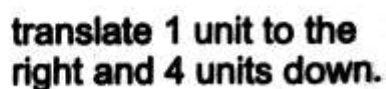
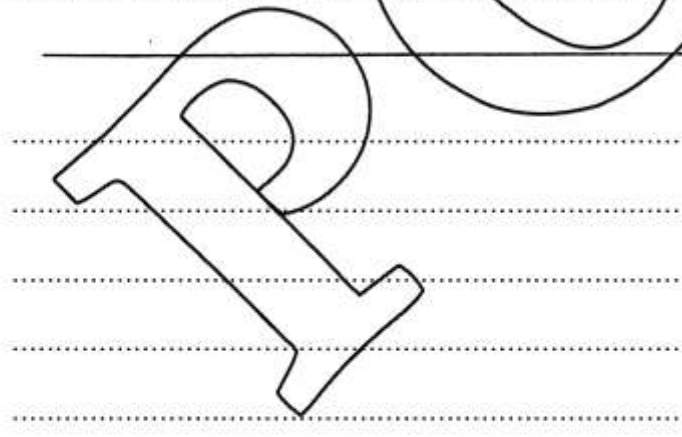
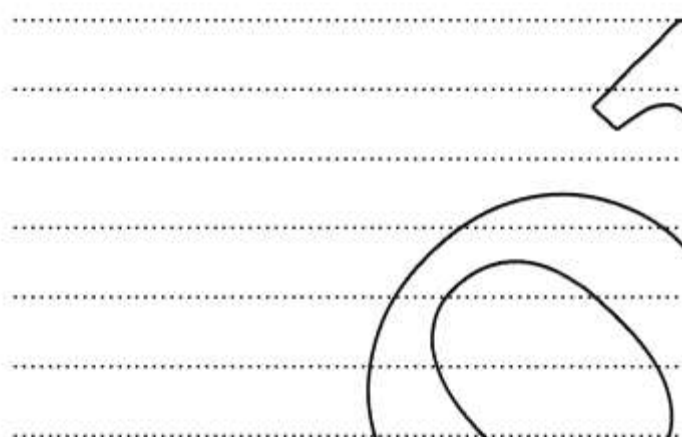
- b** The image of the point A (- 4 , 3) by translation (- 1 , - 4) is .....

[a] (- 5 , - 7)      [b] (- 5 , - 1)      [c] (- 7 , 3)      [d] (- 3 , - 1)

- c** The image of the point (5 , 0) by translation (1 , - 5) is .....

[a] (- 5 , 6)      [b] (6 , - 5)      [c] (0 , 1)      [d] (1 , 0)

- d** The image of the point  $(3, -2)$  by translation  $(-3, 2)$  is .....
- [a]  $(0, 0)$       [b]  $(2, 0)$       [c]  $(3, 0)$       [d]  $(6, 4)$
- e** The image of the point  $(3, -2)$  by translation  $(4, 2)$  is .....
- [a]  $(-7, 0)$       [b]  $(7, 0)$       [c]  $(-1, 4)$       [d]  $(1, 7)$
- f** If  $(x, y)$  is the image of the point  $(3, -2)$  by translation  $(1, 3)$ , then the point  $(x, y) = \dots\dots\dots$
- [a]  $(2, 1)$       [b]  $(2, 4)$       [c]  $(1, 4)$       [d]  $(4, 1)$
- g** The image of the point  $(4, 7)$  by the translation  $(x, y) \longrightarrow (x + 1, y - 2)$  is the point .....
- [a]  $(5, 9)$       [b]  $(3, 5)$       [c]  $(5, 5)$       [d]  $(5, 7)$
- h** The image of the point  $(-1, 2)$  by translation of magnitude of 3 units in the positive direction of the  $x$ -axis is .....
- [a]  $(-1, 5)$       [b]  $(2, 2)$       [c]  $(-2, 2)$       [d]  $(-1, 3)$
- i** The image of the point  $(-3, 4)$  by translation of magnitude of 4 units in the negative direction of the  $y$ -axis is .....
- [a]  $(-3, 0)$       [b]  $(-7, 4)$       [c]  $(-3, 8)$       [d]  $(-1, 4)$
- j** If  $\hat{A}(3, -3)$  is the image of  $A$  by translation  $(x, y) \longrightarrow (x - 1, y - 4)$ , then the point  $A$  is .....
- [a]  $(2, -7)$       [b]  $(4, 1)$       [c]  $(-4, -1)$       [d]  $(2, 1)$
- k** The image of the point  $(2, -1)$  by translation of magnitude 3 units in the positive direction of  $y$ -axis is .....
- [a]  $(2, 2)$       [b]  $(5, -1)$       [c]  $(5, 2)$       [d]  $(2, -4)$
- l** The image of the point  $(3, 0)$  by translation of magnitude 3 units in the negative direction of  $x$ -axis is .....
- [a]  $(0, 0)$       [b]  $(3, 3)$       [c]  $(3, -3)$       [d]  $(0, -3)$

[illegible]



Find the image of each of the following figures by the shown translation under each figure :

.....

.....

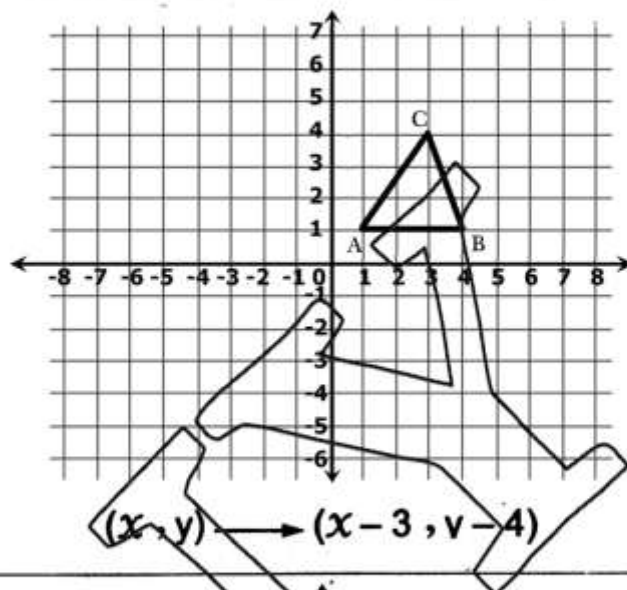
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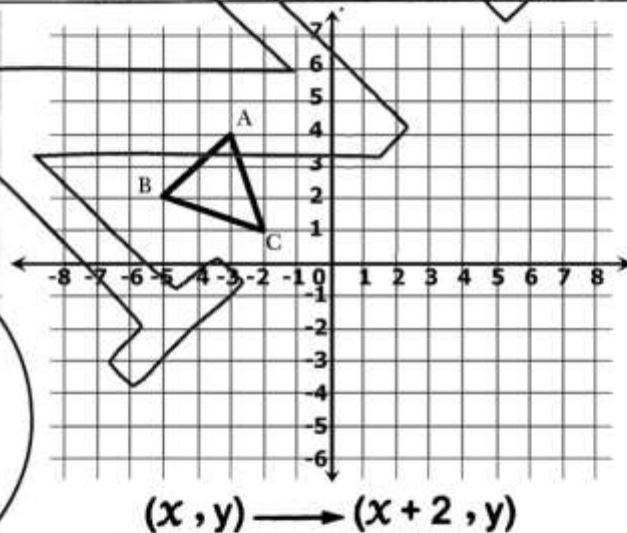
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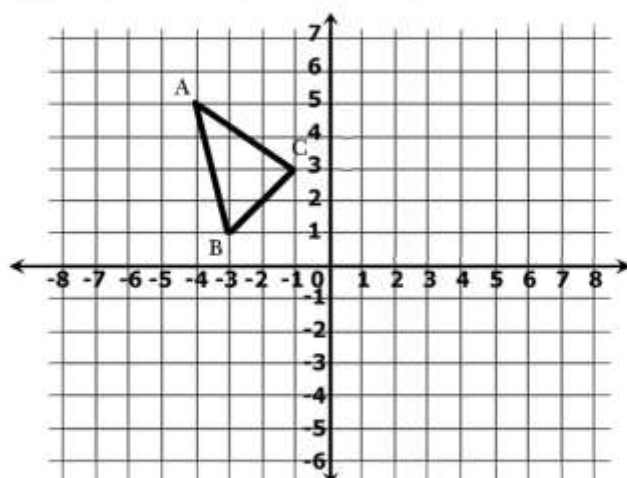
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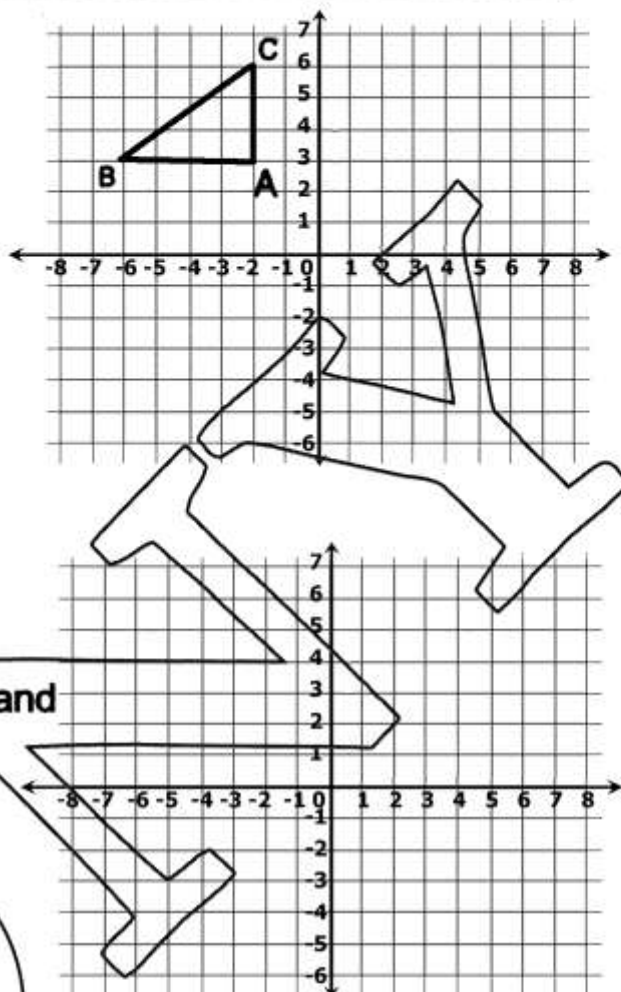
.....



by translation  $(3, -4)$

Copy the graph , then draw the image of the parallelogram ABCD un each of the following translations :

$$(x, y) \longrightarrow (x + 5, y + 2)$$



On the opposite coordinate plane :

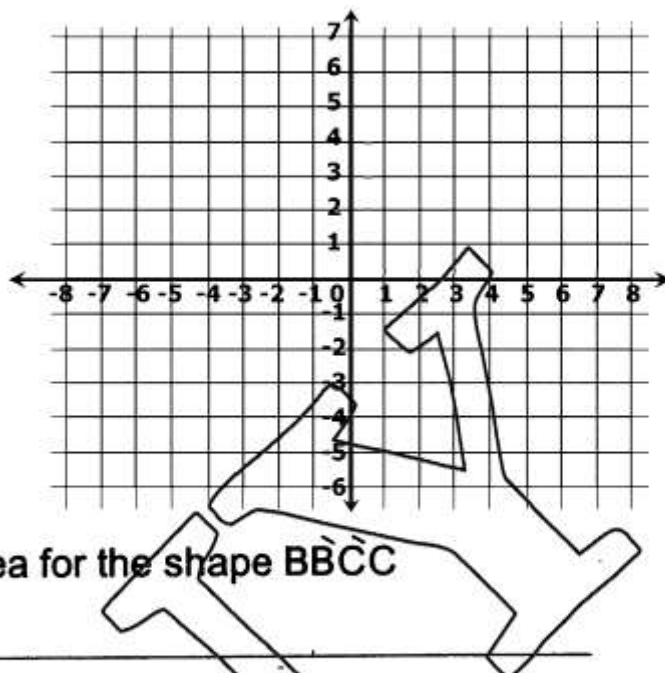
a Determine the following points :  
A (2 , - 2) , B (1 , 1) and C (1 , 6)

b Find  $\hat{A}$  which is the image of the point A by translation (2 , - 1)

c Find  $\overline{B\hat{C}}$  which is the image of  $\overline{BC}$  by translation (3 , 0)

d Find  $\overline{BC}$  and  $\overline{B\hat{C}}$

e Calculate the perimeter and the area for the shape  $B\hat{B}\hat{C}\hat{C}$



Using the geometric tools , draw the image of each of the following :

By translation MN in the direction of  $\overrightarrow{MN}$  as shown in each case.

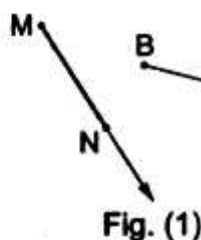


Fig. (1)

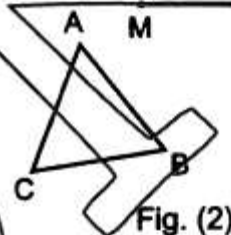
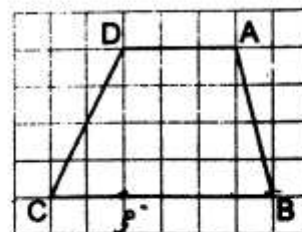
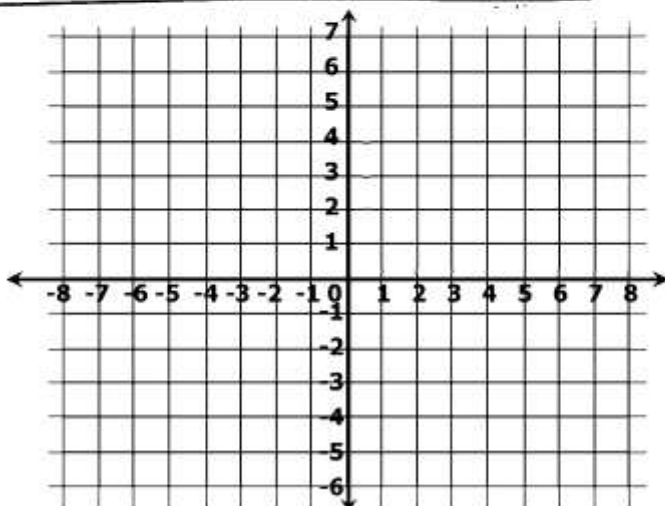


Fig. (2)

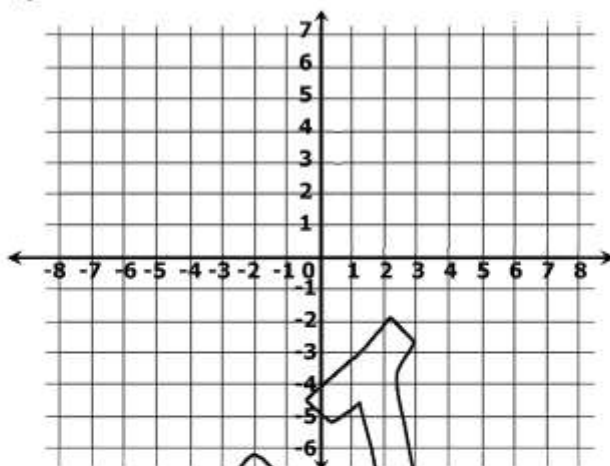
Using the grid , draw the image of the figure ABCD by the translation of 4 units in the direction of  $\overrightarrow{BC}$



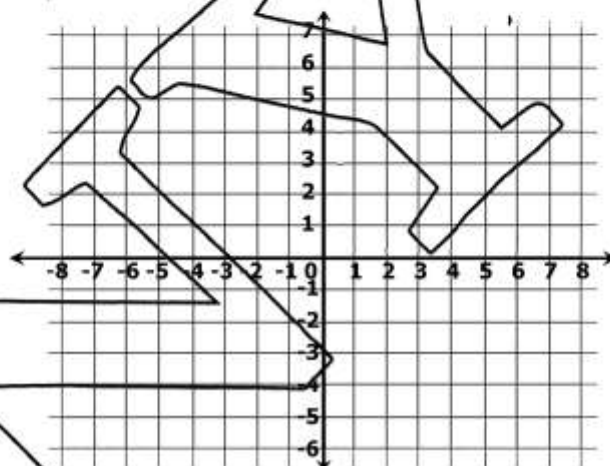
On a square lattice , draw  $\overline{AB}$  where A (2 , 3) and B (4 , 1) , then draw the image of  $\overline{AB}$  by the translation  $(x , y) \longrightarrow (x + 3 , y + 2)$



Draw  $\triangle ABC$ , where  $A(1, 1)$ ,  
 $B(-3, -1)$  and  $C(0, -5)$  then  
 determine graphically its  
 image by translation  $(5, 0)$

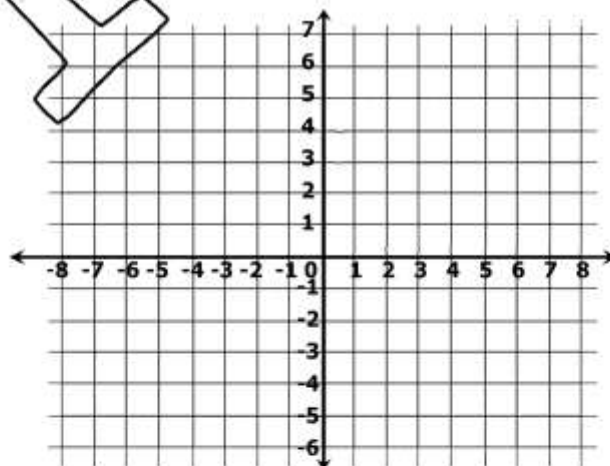


Determine in the coordinate plane  
 the following points  $A(-3, 4)$ ,  
 $B(1, 4)$  and  $C(1, 2)$ , then find :  
 (a)  $AB = \dots\dots\dots$ ,  $BC = \dots\dots\dots$   
 (b) The image of  $\triangle ABC$  by  
 the translation  $(0, -3)$

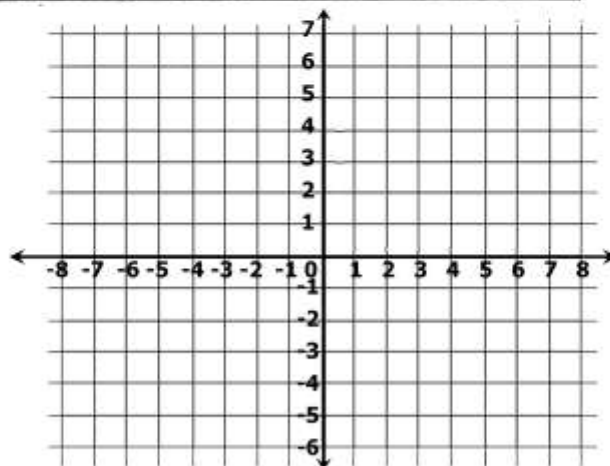


Represent the points  $A(2, 3)$ ,  
 $B(4, 3)$  and  $C(4, 7)$  in the lattice,  
 then find :

- (a)  $BC = \dots\dots\dots$  length unit  
 $AB = \dots\dots\dots$  length unit  
 (b) The image of  $\triangle ABC$  by  
 translation  $(0, -4)$   
 (c) The area of  $\triangle ABC$



The point  $A(3, -3)$  is the image of  
 the point  $A$  by the translation  
 $(x, y) \rightarrow (x - 1, y - 4)$  Locate  $A$ ,  
 then by the same translation,  
 draw the image of  $\triangle ABC$  where  
 $B(5, 0)$  and  $C(-1, -2)$





## Lesson 1

# The set of integers (Z)

(1) Put (  $\checkmark$  ) if the number is an integer :

a- 7 ☐

b-  $\frac{3}{7}$  ☐

c-  $\frac{6}{2}$  ☐

d- - 100 ☐

e- 3.2 ☐

f-  $2\frac{2}{3}$  ☐

\*\*\*\*\*

(2) Put the suitable sign "  $\notin$ ,  $\in$ ,  $\subset$ ,  $\not\subset$  ":

a) -3 ☐ N

b)  $|-20|$  ☐ Z

c) 0 ☐ Z

d) 0 ☐  $Z^+$

e)  $\{1, -2\}$  ☐ N

f)  $\{0\}$  ☐  $Z^-$

g)  $\frac{1}{7}$  ☐ Z

h)  $Z^-$  ☐ Z

\*\*\*\*\*

(3) Write an integer to represent each situation:

a- Karim 's salary is L.E. 350 ( ..... )

b- A temperature is  $12\text{ }^{\circ}\text{C}$  below zero. ( ..... )

c- She's swimming 10 m. deep ( ..... )

d- A temperature is  $5\text{ }^{\circ}\text{C}$  above zero. ( ..... )

e- 3 steps forward. ( ..... )

f- A bank deposit of L.E.750 ( ..... )

g- A weight loss of 6 kg. ( ..... )

**(4) Represent the following numbers on the numberline:**

**a) 3 , -4 , 1 , -2**

**b) -3 , 0 , 2 , 1 , -6 , 5**

\*\*\*\*\*

**(5) Write the opposite ( inverse ) of each integer :**

**a) -3**

**d) -195**

**b) 12**

**e) |-34|**

**c) 0**

\*\*\*\*\*

**(6) Find:**

**a) |-5|**

**b) |-10|**

**c) |-21|**

**d) -|-12|**

**(7) Find each of the following :**

**a) |-3|+|2|**

**b) |5|-|4|**

**c) |-2|+|-13|**

**d) |-100|-|-50|**

e)  $|-4|+|3|$

f)  $|-3|\times|-5|$

g)  $|-10|\times|2|$

\*\*\*\*\*

**(8) Find the value of x :**

a)  $|x|=5$

b)  $|-4|=x$

c)  $|x|=12$

d)  $|3|=x$

\*\*\*\*\*

**(9) Complete:**

a)  $\mathbb{Z} = \dots \cup \dots \cup \dots$

b)  $\mathbb{Z} = \mathbb{N} \cup \dots$

c)  $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots$

d)  $\mathbb{Z} - \mathbb{N} = \dots$

e)  $\mathbb{Z}^+ \cup \{0\} = \dots$

f)  $\mathbb{Z} \cap \mathbb{N} = \dots$

g)  $\mathbb{N} - \mathbb{Z}^+ = \dots$

\*\*\*\*\*

**(10) In each of the following , find the value of x:**

a)  $-4 \in \{7, x, -3\}$

b)  $x \in \{-2\}$

c)  $-5 \in \{-1, 0, -3, x\}$

d)  $x \in \{2, 5, -3\} \cap \{5, -2, -3\}$

## Lesson 2

### ordering and comparing integers

**(1) Put [ < , > or = ] :**

a) 3  -3

c)  $|-4|$    $|0|$

e) 8   $|-8|$

g) -2  0

i)  $-|-4|$   2

b) -8  4

d) -12  -3

f) -3  -4

h)  $|-13|$   3

\*\*\*\*\*

**(2) Complete the following :**

a) The number ..... is neither positive nor negative.

b) The largest negative integer is .....

c) The smallest positive integer is .....

d) The smallest non negative integer is .....

e) The largest non-positive integer is .....

\*\*\*\*\*

**(3) Arrange in an ascending order each of the following:**

a) 3 , -4 , 5 , 1 , -2

b) -22 , 11 , -11 , 0 , 7



**(4) Arrange in a descending order each of the following :**

**a) -9 , 3 , -23 , 6 , -7**

**b) 8 , -13 , -19 , 0 , -15**

**(5) Write using the listing method each of the following sets:**

**a) The set of integers greater than 2**

**b) The set of integers smaller than 0**

**c) The set of integers greater than -3 and smaller than 2**

\*\*\*\*\*

**(10) Complete the following:**

**a) -5 , -4 , -3 , ..... , ..... , .....**

**b) 3 , 2 , 1 , ..... , ..... , .....**

**c) -2 , -4 , -6 , ..... , ..... , .....**

## Lesson 3

### Adding and subtracting integers

The sum of two positive integers is a positive integer.  
The sum of two negative integers is a negative integer.

The sum of a positive integer and a negative integer will have the sign of the integer with the greatest absolute value.

Addition operation is always possible in  $\mathbb{Z}$ .

#### Properties of addition in $\mathbb{Z}$

##### (1) Closure property :

The operation is always possible in  $\mathbb{Z}$  or  $\mathbb{Z}$  is closed under addition.

##### (2) Commutative property:

If  $a, b$  are two integers, then  $a + b = b + a$

##### (3) Associative property :

If  $a, b$  and  $c$  are three integers, then:

$$a + b + c = (a + b) + c = a + (b + c)$$

##### (4) The existence of additive identity ( opposite ) property:

For any integer  $a$ , we have :  $a + 0 = a$

Zero is the additive identity element in  $\mathbb{Z}$

##### (5) The existence of additive inverse ( opposite) property:

For every integer ( $a$ ) there is an additive inverse ( $-a$ )

Where :  $a + (-a) = 0$

- (1)  $\mathbb{Z}$  is closed under subtraction operation.

The result of subtraction any two integers is an integer.

- (1) The subtraction operation in  $\mathbb{Z}$  is not commutative.  
(2) The subtraction operation in  $\mathbb{Z}$  is not associative.

(1) Find the result of the following:

- |                    |                  |
|--------------------|------------------|
| a) $4 + 2$         | b) $9 + (-8)$    |
| c) $(-10) + (-16)$ | d) $(-2) + (-1)$ |
| e) $0 + (-5)$      | f) $-4 + 16$     |
| g) $-5 + 9$        | h) $18 + (-18)$  |

\*\*\*\*\*

(2) Write the property of addition in the set Z in each of the Following:

- |                                    |           |
|------------------------------------|-----------|
| a) $-5 + 3 = 3 + (-5)$             | ( ..... ) |
| b) $6 + (-6) = 0$                  | ( ..... ) |
| c) $0 + (-7) = -7$                 | ( ..... ) |
| d) $(-10 + 5) + 3 = -10 + (5 + 3)$ | ( ..... ) |
| e) $-a + a = 0$                    | ( ..... ) |

\*\*\*\*\*

(2) Complete each of the following:

- a)  $4 + (-3) = (-3) + \dots$
- b)  $(7) + \dots = 3$
- c)  $-7 - (-4) = -7 + \dots$
- d) The additive inverse of 0 is .....
- e) The result of subtracting 7 from  $(-2)$  is .....
- f) The result of subtracting -5 from 3 is .....

g) If  $a + (-3) = b + a$ , then  $b = \dots$

h) If  $a + b = b$ , then  $a = \dots$

i)  $5 + \dots = 0$

j)  $4 + \dots = -3$

k)  $(-7) + \dots = 0$

\*\*\*\*\*

(3) Use the properties of addition in  $\mathbb{Z}$  to find :

a)  $-7 + 2 + (-13)$

b)  $-23 + (-18) + 23$

c)  $5 + (-3) + 7 + (-9)$

\*\*\*\*\*

(4) Find each of the following:

a)  $(-3) + (-4) + 0$

b)  $-3 + 6 + (-2)$

c)  $(-15) + (-10) + (-5)$

# lesson1

## The distance between two points in the coordinate plane

### 1) In the opposite coordinate plane:

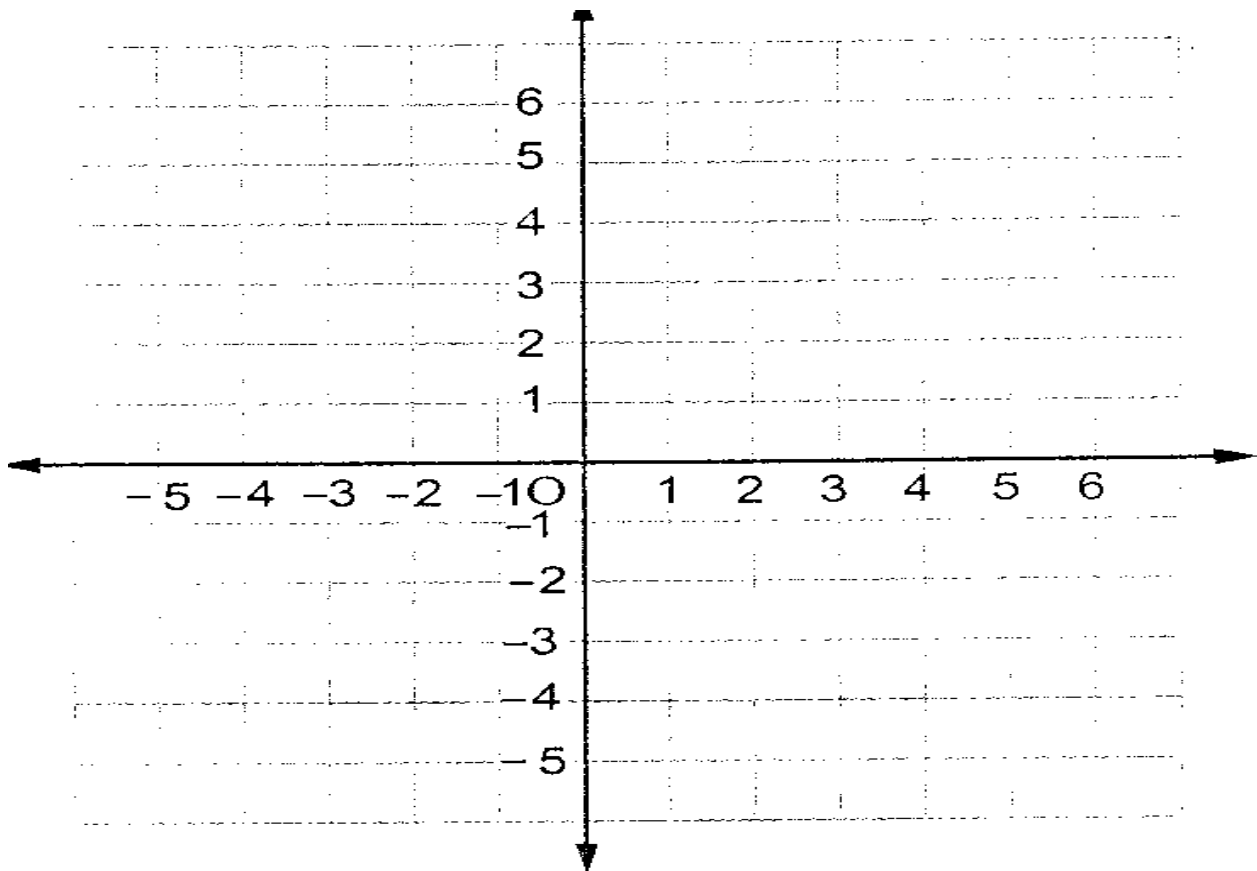
a) Determine the position of the following points:

A(-3, -3) B(-3, 2), C(5, 2) and D(5, -3)

And mention the name of the shape ABCD

b) Find the area and the perimeter of the shape ABCD

---





**2) In the opposite coordinate plane:**

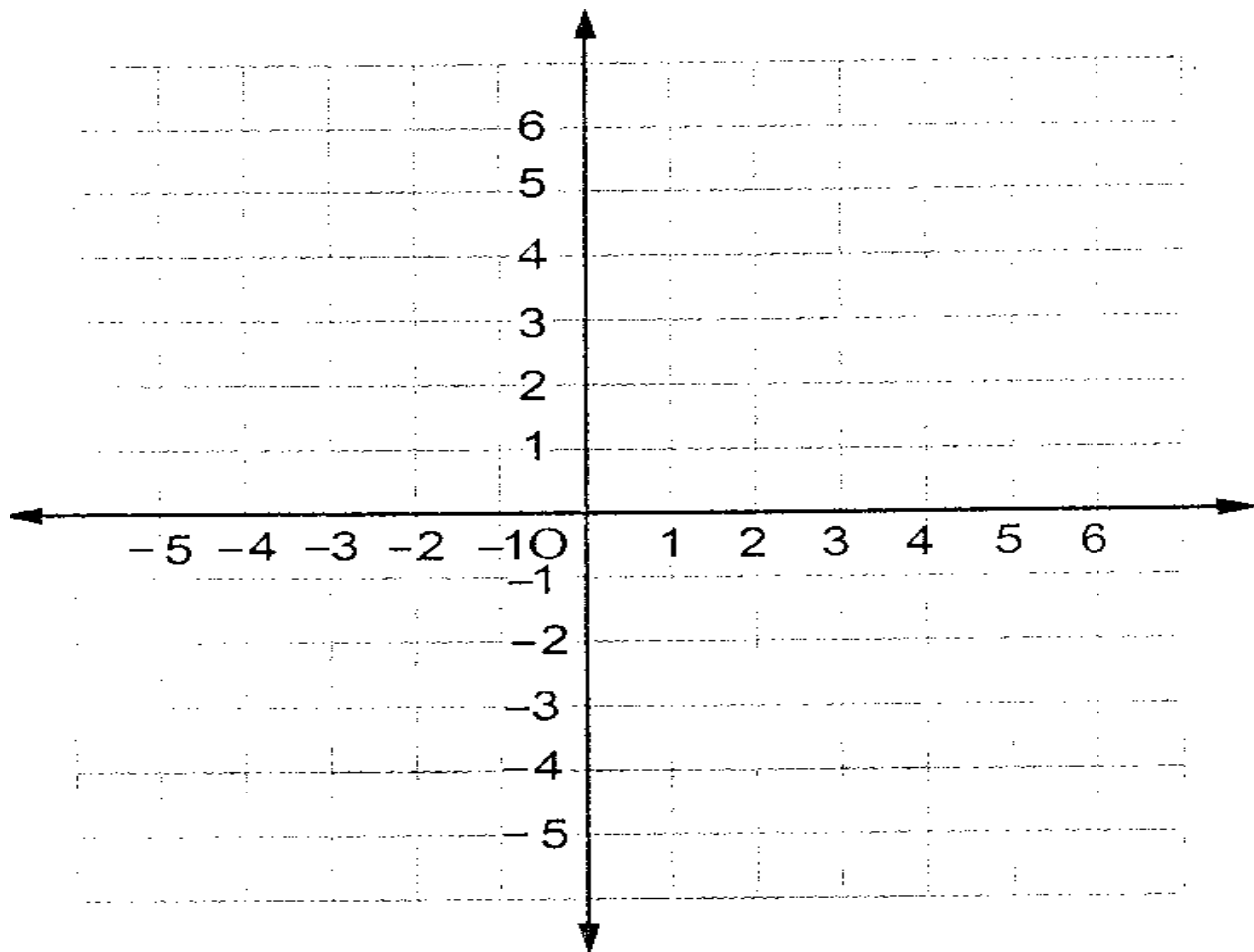
**A) Determine the position of the following points: A (-4 , -4)**

**, B (2 , -4) , C (2 , 2) and D (-4 , 2)**

**And mention the name of the shape ABCD**

**B) Find the perimeter and the area of the shape ABCD**

**C) Determine whether the shape is symmetric or not? Why?**



3) In the opposite coordinate plane:

Determine the position of the following points:

A (-1 , - 4) , B (-1 , 3) and C (5 , -4)

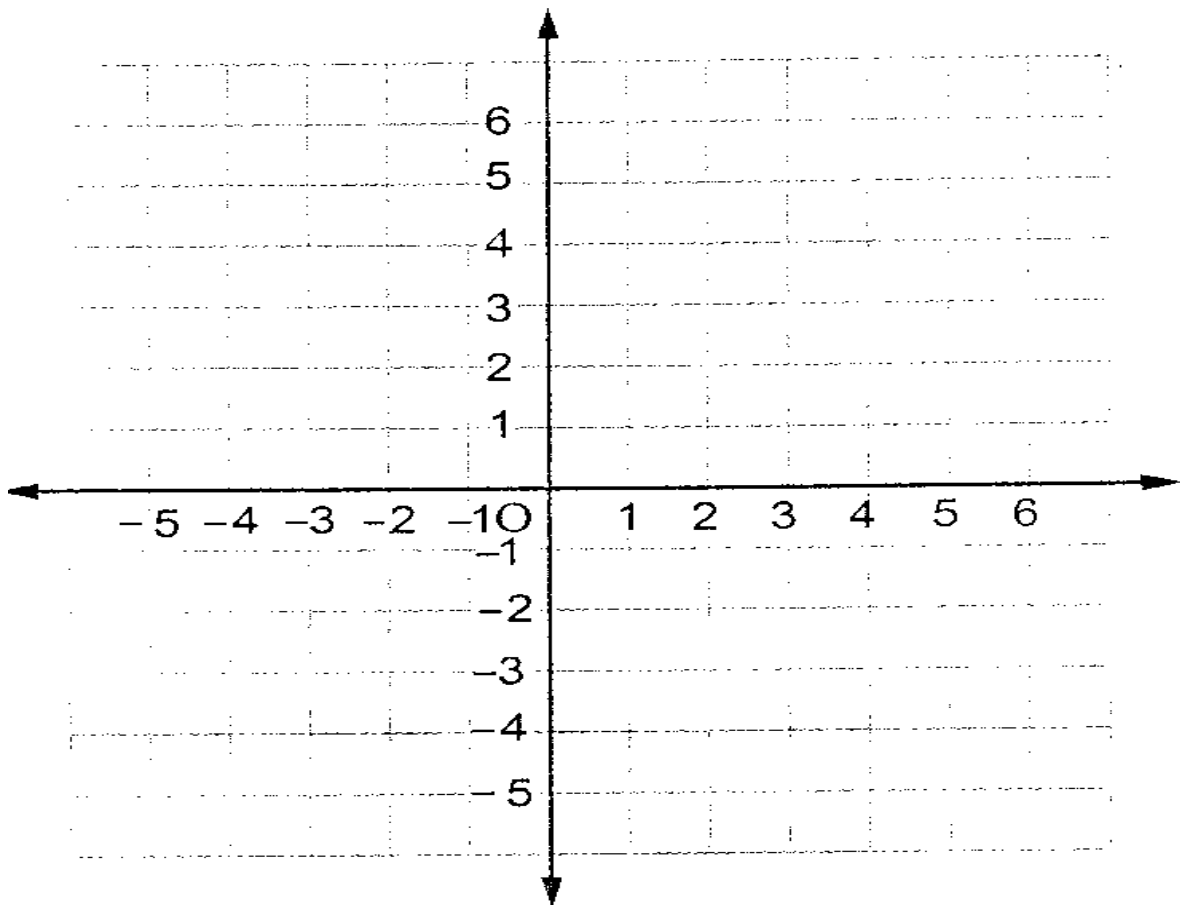
Then find:

a) the length of  $\overline{AB}$  and  $\overline{AC}$

B) The type of the triangle ABC with respect to  
Its sides and angles

c) The area of  $\Delta ABC$

---



**4) In the opposite coordinate plane:**

**ABCD is a square, then complete:**

**a) The coordinate of the points :**

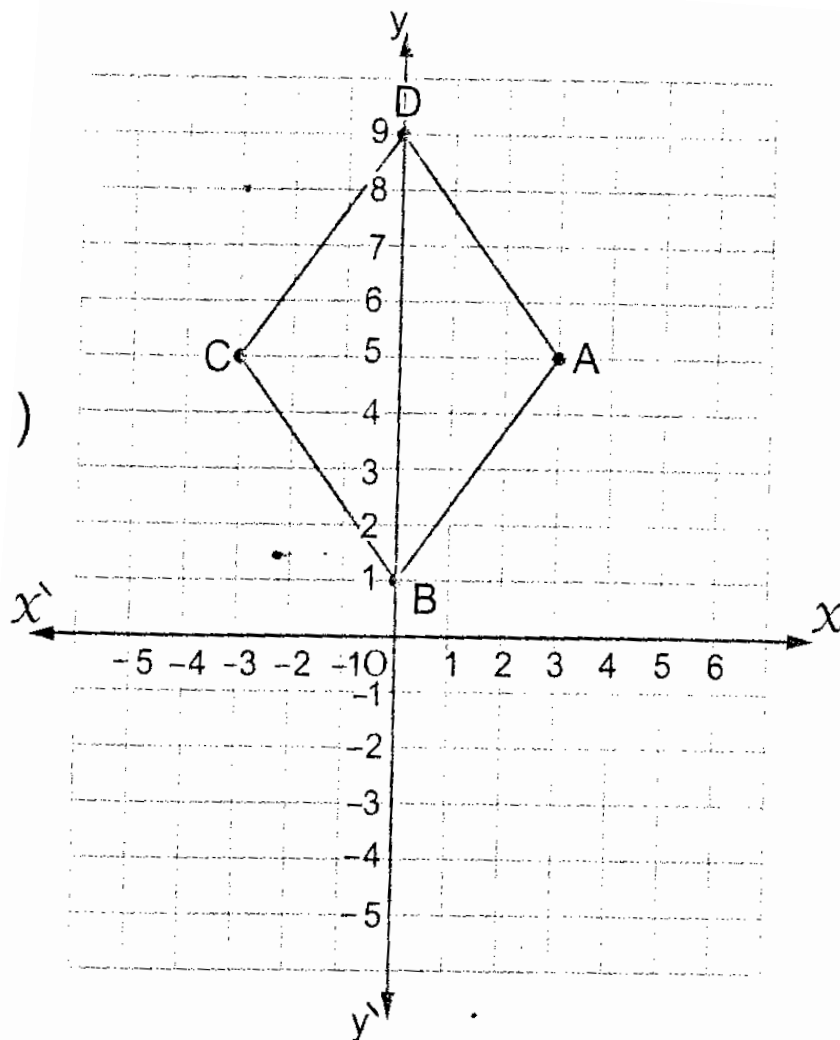
**A ( ..... , ..... ) , B ( ..... , ..... ) , C ( ..... , ..... )**

**And D ( ..... , ..... )**

**B) The length of  $\overline{AC}$  = .....**

**The length of  $\overline{DB}$  = .....**

**C) The area of the square ABCD = .....**



# lesson2

## translation

### 1) Complete each of the following:

1) The image of the point (2 , 5) by translation

$(x, y) \longrightarrow (x + 2, y + 1)$  is .....

2) The image of the point (3 , 2) by translation

$(x, y) \longrightarrow (x + 3, y - 2)$  is .....

3) The image of the point (-5 , 4) by translation

$(x, y) \longrightarrow (x + 4, y - 5)$  is .....

4) The image of the point (-1 , 3) by translation (2 , -3 ) is .....

5) The image of the point (0 , 5 ) by translation ( -2 , 1) is .....

6) The image of the point ( - 2 , - 5) by translation

$(x, y) \longrightarrow (x - 2, y)$  is .....

7) The image of the point (3 , - 2) by translation

$(x, y) \longrightarrow (x, y + 3)$  is .....

8) The image of the point ..... By translation

$(x, y) \longrightarrow (x - 2, y + 3)$  is (7 , 4)

9) If the image of the point (3 , 2) is the point (6 , 1 ) , then the translation

Rule is  $(x, y) \longrightarrow ( \dots\dots\dots, \dots\dots\dots )$

10) The image of the point A (3 , 6) by translation 3 units in the negative

Direction of x – axis is .....

**2) Choose the correct answer:**

- 1) The image of the point A ( 1 , 2 ) by translation ( 1 , -1 ) is .....  
a) ( 2 , 1 )      b) ( 2 , 3 )      c) ( 1 , 1 )      d) ( 1 , 3 )
- 2) The image of the point A ( -4 , 3 ) by translation ( -1 , -4 ) is .....  
a) ( -5 , -7 )      b) ( - 5 , -1 )      c) ( -7 , 3 )      d) ( -3 , -1 )
- 3) The image of the point ( 5 , 0 ) by translation ( 1 , -5 ) is .....  
a) ( -5 , 6 )      b) ( 6 , -5 )      c) ( 0 , 1 )      d) ( 1 , 0 )
- 4) The image of the point ( 3 , -2 ) by translation ( -3 , 2 ) is .....  
a) ( 0 , 0 )      b) ( 2 , 0 )      c) ( 3 , 0 )      d) ( 6 , 4 )
- 5) The image of the point ( 3 , -2 ) by translation ( 4 , 2 ) , then the point ( x , y )  
is .....  
a) ( -7 , 0 )      b) ( ( 7 , 0 )      c) ( - 1 , 4 )      d) ( 1 , 7 )
- 6) If ( x , y ) is the image of the point ( 3 , -2 ) by translation ( 1 , -3 ) , then  
The point ( x , y ) = .....  
a) ( 2 , 1 )      b) ( 2 , 4 )      c) ( 1 , 4 )      d) ( 4 , -5 )
- 7) The image of the point ( 4 , 7 ) by the translation (x+1 , y – 2)  
Is the point .....  
a) ( 5 , 9 )      b) ( 3 , 5 )      c) ( 5 , 5 )      d) ( 5 , 7 )
- 8) The image of the point ( - 1 , 2 ) by translation of magnitude of 3 units in  
The positive direction of the x – axis is .....
- 9) The image of the point ( -3 , 4 ) by translation of magnitude of 4 units in the  
Negative direction of the y – axis is .....



3) On the opposite coordinate plane:

1) Determine the following points :

A ( 2 , -2 ) , B ( 1 , 1 ) and C ( 1 , 6 )

2) Find  $\hat{A}$  which is the image of the point A by translation ( 2 , -1 )

3) Find  $\overline{B'C'}$  which is the image of BC by translation ( 3 , 0 )

4) Find BC and  $\overline{BB'}$

5) Calculate the perimeter and the area for the shape  $BB'C'C$

---

4) Using the square lattice ,draw  $\Delta OBC$  where O is the origin point , B (3,0)  
C (0,-2) then draw its image by the translation  $(x,y) \longrightarrow (x-4, y+1)$

---

5) Draw  $\Delta ABC$  where A (1,1) , B (-3 , -1) and C (0,-5) then determine graphically Its image by translation (5 , 0)

---

6) Find the image of each of the following points by the translation

$(x,y) \longrightarrow (x+2, y-3)$  following by translation

$(x, y) \longrightarrow (x-3, y+1)$

a) (4 , -2)

b) (-1 , 3)

c) (0 , 2)

**Choose the correct answer:**

1	$ -5  = \dots\dots$ <b>(a)</b> -5 <b>(b)</b> 5 <b>(c)</b> 10 <b>(d)</b> 0
2	$\frac{15}{5} \dots\dots \mathbb{Z}$ <b>(a)</b> $\in$ <b>(b)</b> $\notin$ <b>(c)</b> $\subset$ <b>(d)</b> $\not\subset$
3	An integer included between -5 and 3 is ..... <b>(a)</b> 4 <b>(b)</b> 1 <b>(c)</b> -6 <b>(d)</b> -7
4	$3 +  -3  = \dots\dots$ <b>(a)</b> 0 <b>(b)</b> 6 <b>(c)</b> 3 <b>(d)</b> -3
5	$\mathbb{Z} - \mathbb{Z}^- = \dots\dots$ <b>(a)</b> $\mathbb{Z}^-$ <b>(b)</b> $\emptyset$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\mathbb{Z}^+$
6	$\mathbb{Z} - \mathbb{N} = \dots\dots$ <b>(a)</b> $\mathbb{Z}^-$ <b>(b)</b> $\emptyset$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\mathbb{Z}^+$
7	$7 +  -3  = \dots\dots$ <b>(a)</b> 2 <b>(b)</b> 10 <b>(c)</b> 4 <b>(d)</b> -4
8	$2 \dots\dots \mathbb{Z}$ <b>(a)</b> $\in$ <b>(b)</b> $\notin$ <b>(c)</b> $\subset$ <b>(d)</b> $\not\subset$
9	If $ X  = 7$ , then $X = \dots\dots$ or ..... <b>(a)</b> 5, -5 <b>(b)</b> 4, -4 <b>(c)</b> 3, -3 <b>(d)</b> 7, -7
10	$\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots$ <b>(a)</b> $\mathbb{Z}$ <b>(b)</b> $\emptyset$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\mathbb{Z}^+$
11	$-7 \dots\dots \mathbb{Z}^+$ <b>(a)</b> $\in$ <b>(b)</b> $\notin$ <b>(c)</b> $\subset$ <b>(d)</b> $\not\subset$

12	The number of integers between -2 and 2 is .....
	<b>(a)</b> 0 <b>(b)</b> 1 <b>(c)</b> 2 <b>(d)</b> 3
13	$\{-9\}$ ..... $\mathbb{Z}$
	<b>(a)</b> $\in$ <b>(b)</b> $\notin$ <b>(c)</b> $\subset$ <b>(d)</b> $\not\subset$
14	$\mathbb{Z}^+ \cup \{0\} = \dots\dots$
	<b>(a)</b> $\mathbb{Z}^-$ <b>(b)</b> $\emptyset$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\mathbb{Z}$
15	The greatest non-positive integer is .....
	<b>(a)</b> 0 <b>(b)</b> 1 <b>(c)</b> -1 <b>(d)</b> otherwise
16	The greatest negative integer is .....
	<b>(a)</b> 0 <b>(b)</b> 1 <b>(c)</b> -1 <b>(d)</b> -100
17	$\mathbb{Z}^+ \cup \dots\dots = \mathbb{N}$
	<b>(a)</b> $\mathbb{Z}^-$ <b>(b)</b> $\{0\}$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\emptyset$
18	The smallest positive integer is .....
	<b>(a)</b> 0 <b>(b)</b> 1 <b>(c)</b> -1 <b>(d)</b> 100
19	$\mathbb{Z}^- \cup \mathbb{N} = \dots\dots$
	<b>(a)</b> $\mathbb{Z}$ <b>(b)</b> $\emptyset$ <b>(c)</b> $\mathbb{N}$ <b>(d)</b> $\mathbb{Z}^+$
20	The additive identity element in $\mathbb{Z}$ is .....
	<b>(a)</b> 0 <b>(b)</b> 1 <b>(c)</b> 2 <b>(d)</b> 3
21	$6 + (-10) = \dots\dots$
	<b>(a)</b> 4 <b>(b)</b> 8 <b>(c)</b> -8 <b>(d)</b> -4
22	$(-7) + 3 > \dots\dots$
	<b>(a)</b> -5 <b>(b)</b> -4 <b>(c)</b> 2 <b>(d)</b> 0
23	$3 + (-11) = \dots\dots$
	<b>(a)</b> 4 <b>(b)</b> 8 <b>(c)</b> -8 <b>(d)</b> -4
24	The additive inverse of -5 is .....
	<b>(a)</b> 5 <b>(b)</b> 7 <b>(c)</b> 9 <b>(d)</b> 11
25	$-10 + 7 = \dots\dots$
	<b>(a)</b> 3 <b>(b)</b> 17 <b>(c)</b> -3 <b>(d)</b> -17

26	The additive inverse of 2 is .....
	<b>a</b> -2 <b>b</b> -4 <b>c</b> -6 <b>d</b> -12
27	$ -5  + \dots = 0$
	<b>a</b> -5 <b>b</b> -10 <b>c</b> 0 <b>d</b> 5
28	The additive inverse of 0 is .....
	<b>a</b> 0 <b>b</b> 12 <b>c</b> -1 <b>d</b> 1
29	The image of the point (5 , 1) by the translation (x-1 , y+2) is .....
	<b>a</b> (4 , 3) <b>b</b> (3 , 4) <b>c</b> (2 , 5) <b>d</b> (1 , 6)
30	The image of the point (5 , 3) by the translation (x+1 , y+2) is .....
	<b>a</b> (3 , 2) <b>b</b> (4 , 3) <b>c</b> (5 , 4) <b>d</b> (6 , 5)
31	The image of the point (3 , -3) by the translation (-1 , -2) is .....
	<b>a</b> (4 , -3) <b>b</b> (3 , -4) <b>c</b> (2 , -5) <b>d</b> (1 , -6)
32	The image of the point (5 , 1) by the translation (-1 , 2) is .....
	<b>a</b> (4 , 3) <b>b</b> (3 , 4) <b>c</b> (2 , 5) <b>d</b> ((1 , 6)
33	The image of the point (3 , 1) by the translation 5 units in the negative direction of x-axis is .....
	<b>a</b> (8 , 6) <b>b</b> (8 , 1) <b>c</b> (-2 , 1) <b>d</b> (3 , 6)
34	$-5 + 15 = \dots$
	<b>a</b> -10 <b>b</b> 20 <b>c</b> 10 <b>d</b> -20
35	The set of integers that are grater than -5 = .....
	<b>a</b> {-6, -7, ... } <b>b</b> {-4, -3, ... } <b>c</b> {-4, -3, -2} <b>d</b> {-6, -7, -8}
36	If A( 0 , 1 ) and B( 0 , 5 ), then AB = ..... length units.
	<b>a</b> 1 <b>b</b> 2 <b>c</b> 3 <b>d</b> 4
37	If A( -2 , 2 ) and B( -3 , 2 ), then AB = ..... length units.
	<b>a</b> 1 <b>b</b> 2 <b>c</b> 3 <b>d</b> 4
38	If A( 0 , 1 ) and B( 0 , 3 ), then AB = ..... length units.
	<b>a</b> 1 <b>b</b> 2 <b>c</b> 3 <b>d</b> 4

## Essay Problems:

1 Order the following integers in an ascending order:  
 $5, -2, 1, -10, 3$   
The ascending order is: ..... , ..... , ..... , ..... , .....

2 If  $A = 2$  and  $B = -5$ , find the value of:  
(a)  $A + B =$  .....  
(b)  $A - B =$  .....

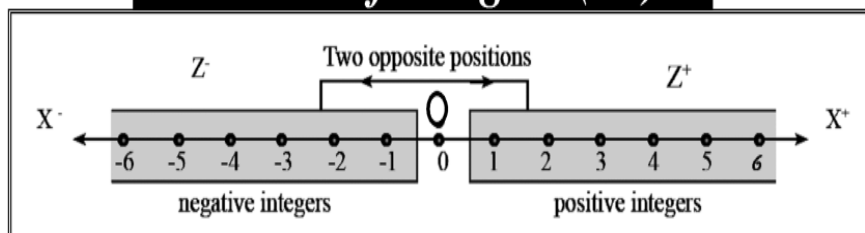
3 Use the properties of addition in  $\mathbb{Z}$  to find the value of:  
 $12 + 36 + (-12) =$  .....  
 $=$  .....  
 $=$  .....  
 $=$  .....

4 Use the properties of addition in  $\mathbb{Z}$  to find the value of:  
 $9 + (-7) + 6 + (-10) =$  .....  
 $=$  .....  
 $=$  .....  
 $=$  .....





## The Set of Integers ( $Z$ )



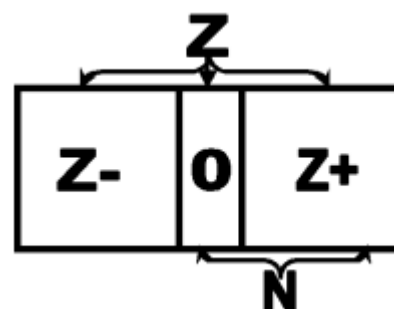
The set of integers is an infinite set and extends without limit from both sides  
Zero is neither positive nor negative .

$$N \subset Z, Z^+ \subset Z, Z^- \subset Z, \{0\} \subset Z$$

The Set Positive Integers:  $Z^+ = \{1, 2, 3, 4, 5, \dots\}$

The Set Negative Integers:  $Z^- = \{-1, -2, -3, -4, -5, \dots\}$

The Set of Integers  $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$



### The absolute value of the integer

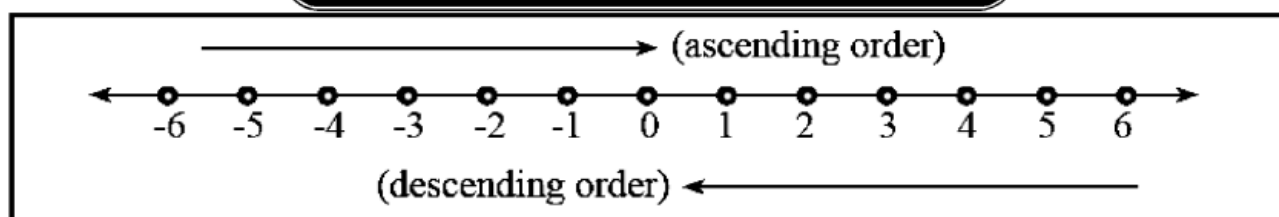
The absolute value of the integer (a) is the distance between the location of (a) and the location of Zero on the number line .

It is always positive and denoted by the symbol  $|a|$

ex

Find the absolute value of : 4 , -4 , 8 , -8  
the solution :  $|4| = 4$  ,  $|-4| = 4$  ,  $|8| = 8$  ,  $|-8| = 8$

### Ordering and Comparing Integers



This means that :

.....  $-3 < -2 < -1 < 0 < 1 < 2 < 3 < \dots$  (ascending order).

.....  $3 > 2 > 1 > 0 > -1 > -2 > -3 \dots$  (descending order).

ex

Arrange the following numbers in an ascending order :

-1 , 3 , 1 , -5 , 7

The ascending order is : -5 , -1 , 1 , 3 , 7

ex

Put the correct sign (>, < or =) :

(a)  $-7 > -9$

(b)  $3 > -13$

(c)  $-4 < 0$

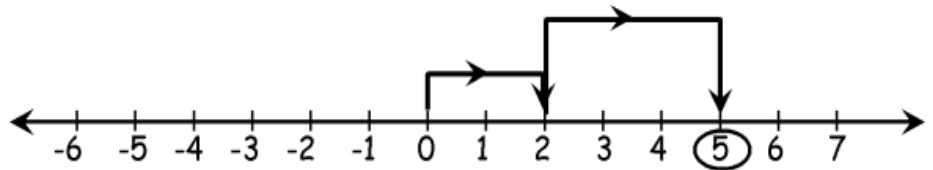
(d)  $|-11| = 11$

(e)  $-7 < -|-5|$

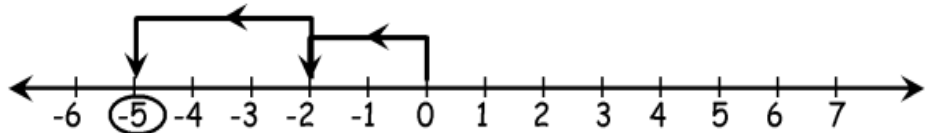
(f)  $30 < 103$

## Adding and subtracting Integers

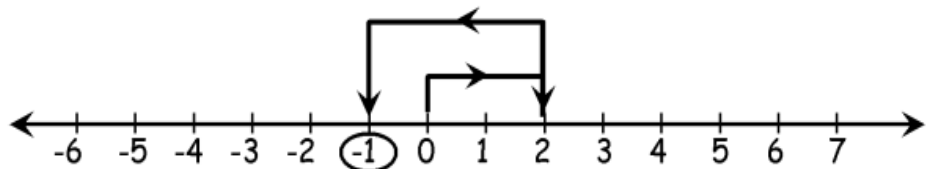
$$2 + 3 = 5$$



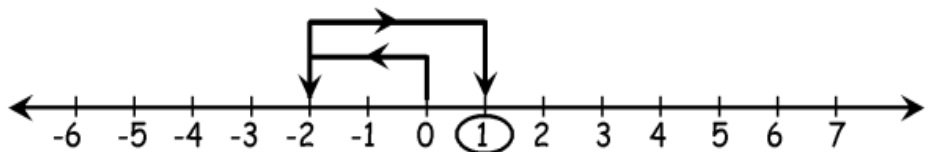
$$(-2) + (-3) = (-5)$$



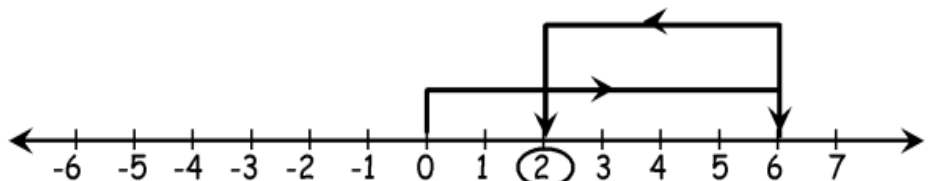
$$2 + (-3) = (-1)$$



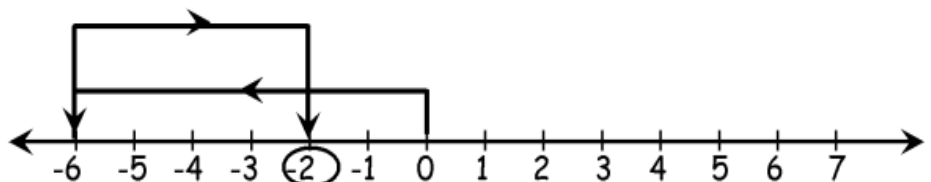
$$(-2) + 3 = 1$$



$$6 + (-4) = 2$$



$$(-6) + 4 = (-2)$$



**i.e.**

Adding positive integers and adding natural numbers are the same.  
 Adding two negative integers = negative integer  
 The sum of two different integers = positive or negative integer.

## Properties of addition operation in $\mathbb{Z}$ :

- 1- **Closure property** :  $\mathbb{Z}$  is closed under the addition operation,  
this means that the sum of any two integers is an integer.

$$\text{If } a \in \mathbb{Z}, b \in \mathbb{Z} \text{ Then } a + b = c \in \mathbb{Z}$$

ex

This means that : the addition operation is always possible in  $\mathbb{Z}$

$$\begin{array}{lll} 3 \in \mathbb{Z}, & 8 \in \mathbb{Z} & 3 + 5 = 8 \in \mathbb{Z} \\ -3 \in \mathbb{Z}, & -8 \in \mathbb{Z} & (-3) + (-5) = -8 \in \mathbb{Z} \\ -3 \in \mathbb{Z}, & 8 \in \mathbb{Z} & (-3) + 5 = 2 \in \mathbb{Z} \end{array}$$

- 2 - **Commutative property** :

The sum of any two integers doesn't change when commutating their positions.

$$\text{If } a, b \text{ are two integers, then : } a + b = b + a$$

ex

$$\begin{array}{lll} \therefore 3 + 5 = 8 \text{ \& } 5 + 3 = 8 & \therefore 3 + 5 = 5 + 3 = 8 \\ \therefore (-5) + (-3) = (-8) \text{ \& } (-3) + (-5) = (-8) & \therefore (-5) + (-3) = (-3) + (-5) = (-8) \\ \therefore (-5) + 3 = (-2) \text{ \& } 3 + (-5) = (-2) & \therefore (-5) + 3 = 3 + (-5) = (-2) \end{array}$$

- 3- **The additive - identity** :

Zero is the additive identity (neutral) in  $\mathbb{Z}$  as It was in  $\mathbb{N}$ .

$$\text{If } a \text{ is an integer, then : } a + 0 = 0 + a = a$$

ex

$$\begin{array}{lll} \therefore 3 + 0 = 3 \text{ \& } 0 + 3 = 3 & \therefore 3 + 0 = 0 + 3 = 3 \\ \therefore 0 + (-3) = (-3) \text{ \& } (-3) + 0 = (-3) & \therefore 0 + (-3) = (-3) + 0 = (-3) \end{array}$$

- 4- **The additive - inverse** :

for each positive integer (a) on the number line,  
there is an opposite negative integer (-a), where their sum = 0.

ex

$$a + (-a) = (-a) + a = 0$$

$$\therefore 3 + (-3) = 0 \text{ \& } (-3) + 3 = 0 \quad \therefore 3 + (-3) = (-3) + 3 = 0$$

- 5- **Associative property** :

The addition operation is associative in  $\mathbb{Z}$  as it was in  $\mathbb{N}$ .

$$\begin{array}{l} \text{This means : If } a, b, c \text{ are integers} \\ \text{Then : } a + b + c = (a + b) + c = a + (b + c) \end{array}$$

ex

Use the properties of addition operation in  $\mathbb{Z}$  to find the result of  
( -17 , + 19 , + 17 ) State the property used in each step.

$$-17 + 19 + 17$$

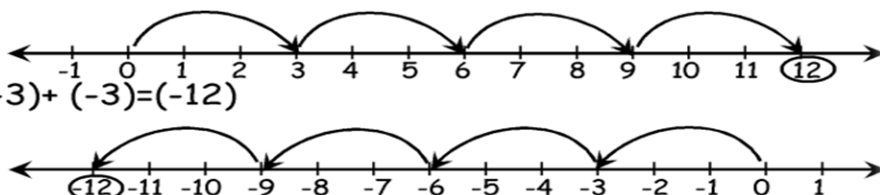
$$\begin{array}{ll} = -17 + (19 + 17) & \text{(Associative)} \\ = -17 + (17 + 19) & \text{(Commutative)} \\ = (-17 + 17) + 19 & \text{(Associative)} \\ = 0 + 19 & \text{(Additive inverse)} \\ = 19 & \text{(additive identity)} \end{array}$$

## Multiplying and dividing integers

### Multiplying Integers :

$$3 \times 4 = 3+3+3+3 = 12$$

$$(-3) \times 4 = (-3) + (-3) + (-3) + (-3) = (-12)$$



The product of two positive integers = positive integer.  
 The product of two negative integers = positive integer  
 The product of two integers having different signs = negative integer

$$\oplus \times \oplus = \oplus \quad \ominus \times \ominus = \oplus \quad \ominus \times \oplus = \ominus \quad \oplus \times \ominus = \ominus$$

ex

find the result of each of the following :

(a)  $5 \times 6 = 30$

(b)  $(-5) \times (-6) = 30$

(c)  $(-5) \times 6 = -30$

(d)  $-5 \times (-6) = -30$

### Properties of multiplication operation on $\mathbb{Z}$ :

**1- Closure property :**  $\mathbb{Z}$  is closed under multiplication operation,  
 this means that the product of any two integers is an integer.  
 i.e. the multiplication operation is always possible in  $\mathbb{Z}$ .

$$\text{If } a \in \mathbb{Z}, b \in \mathbb{Z} \text{ Then } a \times b = c \in \mathbb{Z}$$

ex

This means that : the addition operation is always possible in  $\mathbb{Z}$

$$3 \in \mathbb{Z}, 8 \in \mathbb{Z}$$

$$3 \times 5 = 15 \in \mathbb{Z}$$

$$-3 \in \mathbb{Z}, -8 \in \mathbb{Z}$$

$$(-3) \times (-5) = 15 \in \mathbb{Z}$$

$$-3 \in \mathbb{Z}, 8 \in \mathbb{Z}$$

$$(-3) \times 5 = -15 \in \mathbb{Z}$$

**2- Commutative property :** The multiplication operation is commutative in  $\mathbb{Z}$ .

$$\text{If } a \in \mathbb{Z}, b \in \mathbb{Z} \text{ Then } a \times b = b \times a$$

ex

$$\therefore 3 \times 5 = 15 \text{ \& } 5 \times 3 = 15$$

$$\therefore 3 \times 5 = 5 \times 3 = 15$$

$$\therefore (-5) \times (-3) = 15 \text{ \& } (-3) \times (-5) = 15$$

$$\therefore (-5) \times (-3) = (-3) \times (-5) = 15$$

$$\therefore (-5) \times 3 = (-15) \text{ \& } 3 \times (-5) = (-15)$$

$$\therefore (-5) \times 3 = 3 \times (-5) = (-15)$$

### 3- The multiplicative identity :

one is the multiplicative identity (neutral) in  $\mathbb{Z}$  as It was in  $\mathbb{N}$ .

$$\text{If } a \in \mathbb{Z}, \text{ then } a \times 1 = 1 \times a = a.$$

ex

$$\therefore 3 \times 1 = 3 \text{ \& } 1 \times 3 = 3$$

$$\therefore 3 \times 1 = 1 \times 3 = 3$$

$$\therefore 1 \times (-3) = (-3) \text{ \& } (-3) \times 1 = (-3)$$

$$\therefore 1 \times (-3) = (-3) \times 1 = (-3)$$

### 4- Associative property :

the multiplication operation is associative in  $\mathbb{Z}$  as it was in  $\mathbb{N}$ .

$$a \times b \times c = (a \times b) \times c = a \times (b \times c)$$

ex

$$(-6 \times 8) \times -5 = -48 \times -5 = 240$$

$$-6 \times (8 \times -5) = -6 \times -40 = 240$$

$$\text{i.e. } -6 \times 8 \times -5 = -6 \times (8 \times -5) = (-6 \times 8) \times -5 = 240$$

### 5- The distribution :

It means distributing multiplication operation over addition operation.

$$\text{If } a, b, c \in \mathbb{Z}, \text{ then : } a \times (b + c) = a \times b + a \times c$$

ex

$$5 \times (-3 + 7)$$

$$= 5 \times 4$$

$$= 20$$

$$5 \times -3 + 5 \times 7$$

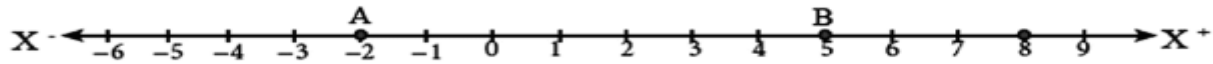
$$= -15 + 35$$

$$= 20$$

$$\text{i.e. } 5 \times (-3 + 7) = 5 \times -3 + 5 \times 7 = 20$$

the distance between two points on a straight line.

$$= | \text{number of the ending point} - \text{number of the starting point} |$$



From the opposite figure :

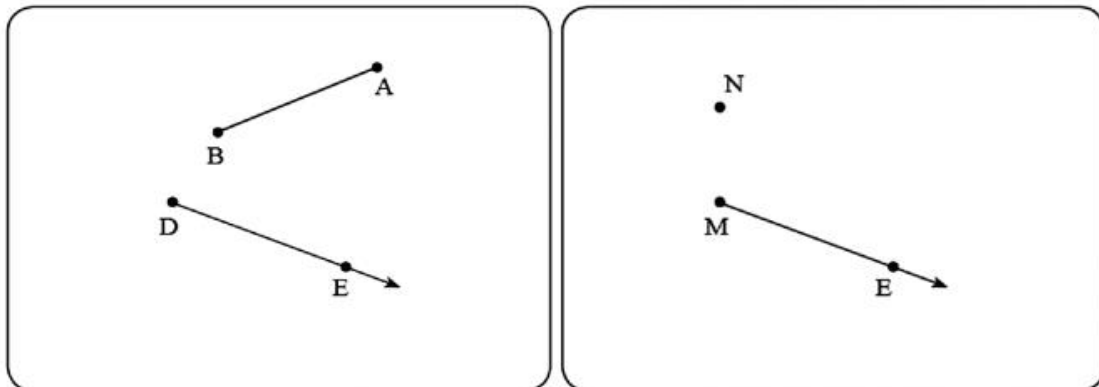
$$AB = | B - A | = | 5 - (-2) | = | 5 + 2 | = 7 \text{ units.}$$

The geometric transformation transforms each point in the plane into a point  $A'$  in the same plane. Also

**First : Translation of a point in the plane.**

Find the following :

- The image of the point N by translation  $\overline{ME}$  in the direction of  $\overrightarrow{ME}$ .
- The image of the  $\overline{AB}$  by translation its magnitude is 3 cm in the direction of  $\overrightarrow{DE}$ .



**Third : The translation of a geometric shape in the coordinate plane**

**Example**

In the opposite figure, Find the image of the  $\triangle ABC$  where :

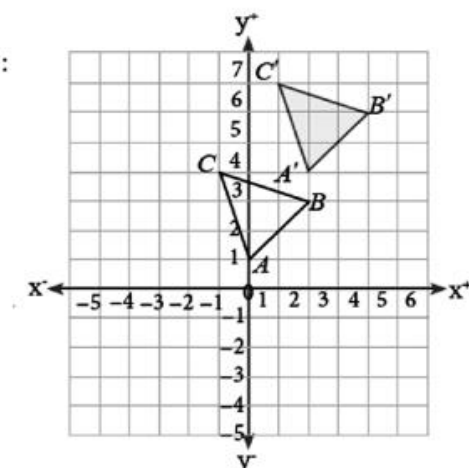
$A(0, 1)$ ,  $B(2, 3)$  and  $C(-1, 4)$  by translation  $(X + 2, y + 3)$ .

**Solution :**

$$A' = (0 + 2, 1 + 3) = (2, 4)$$

$$B' = (2 + 2, 3 + 3) = (4, 6)$$

$$C' = (-1 + 2, 4 + 3) = (1, 7)$$





# 1 Choose the correct Answer

1.	$ -5  =$ ..... A) -5                      B) 5                      C) 10                      D) 0
2.	$\frac{15}{5}$ ..... $\mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$
3.	If $ X  = 4$ , then $X =$ ..... or ..... A) 5 , -5                      B) 4 , -4                      C) 3 , -3                      D) 7 , -7
4.	The integer included between -5 and 3 is ..... A) 4                      B) 1                      C) -6                      D) -7
5.	$\mathbb{Z} = \mathbb{Z}^+ \cup$ ..... $\cup$ ..... A) $\mathbb{Z}^- , \{0\}$ B) $\mathbb{Z}^+ , \{0\}$ C) $\{0\} , \mathbb{N}$ D) $\mathbb{N} , \{0\}$
6.	$ -7  =$ ..... A) -7                      B) 7                      C) 8                      D) -8
7.	$3 +  -3  =$ ..... A) 0                      B) 6                      C) 3                      D) -3
8.	If $ X  = 5$ , then $X =$ ..... or ..... A) 5 , -5                      B) 4 , -4                      C) 3 , -3                      D) 7 , -7
9.	$\mathbb{Z} - \mathbb{Z}^- =$ ..... A) $\mathbb{Z}^-$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
10.	$ -1  =$ ..... A) -1                      B) 1                      C) 10                      D) 0
11.	$5 +  -5  =$ ..... A) 0                      B) 5                      C) 10                      D) -10
12.	$\mathbb{Z} - \mathbb{N} =$ ..... A) $\mathbb{Z}^-$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
13.	$7 +  -3  =$ ..... A) 2                      B) 10                      C) 4                      D) -4
14.	2 ..... $\mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$

15.	If $ X  = 7$ , then $X =$ ..... or ..... A) 5, -5      B) 4, -4      C) 3, -3      D) 7, -7
16.	$\mathbb{Z} - \mathbb{Z}^+ =$ ..... A) $\mathbb{Z}^-$ B) $\{0\} \cup \mathbb{Z}^-$ C) $\mathbb{N}$ D) $\{0\} \cup \mathbb{Z}^+$
17.	$ \frac{5-8}{3}  =$ ..... A) 1      B) 6      C) -6      D) -2
18.	$9 +  -2  =$ ..... A) 7      B) 11      C) 9      D) -11
19.	$5$ ..... $\mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$
20.	The number of integers between -1 and 1 is ..... A) 0      B) 1      C) 2      D) 3
21.	$\mathbb{Z}^+ \cap \mathbb{Z}^- =$ ..... A) $\mathbb{Z}$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
22.	$ \frac{3-8}{5}  =$ ..... A) 1      B) 6      C) -6      D) -2
23.	$-7$ ..... $\mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$
24.	The number of integers between -2 and 2 is ..... A) 0      B) 1      C) 2      D) 3
25.	$\mathbb{Z}^+ - \mathbb{Z}^- =$ ..... A) $\mathbb{Z}^-$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
26.	The number of integers between -3 and 3 is ..... A) 3      B) 4      C) 5      D) 6
27.	$\mathbb{Z}^- - \mathbb{Z}^+ =$ ..... A) $\mathbb{Z}^-$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
28.	$\{-9\}$ ..... $\mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$

29.	$\mathbb{Z}^+ \cup \{0\} = \dots\dots\dots$ A) $\mathbb{Z}^-$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
30.	$ -3  +  -2  = \dots\dots\dots$ A) 1                      B) 3                      C) 5                      D) 2
31.	$\{-2\} \dots\dots\dots \mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$
32.	The greatest negative integer is $\dots\dots\dots$ A) 0                      B) 1                      C) -1                      D) Otherwise
33.	$\mathbb{Z}^+ \cup \dots\dots\dots = \mathbb{N}$ A) $\mathbb{Z}^-$ B) $\{0\}$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
34.	$\{5\} \dots\dots\dots \mathbb{Z}$ A) $\in$ B) $\notin$ C) $\subset$ D) $\not\subset$
35.	The smallest positive integer is $\dots\dots\dots$ A) 0                      B) 1                      C) -1                      D) Otherwise
36.	$\mathbb{Z}^- \cup \mathbb{N} = \dots\dots\dots$ A) $\mathbb{Z}$ B) $\emptyset$ C) $\mathbb{N}$ D) $\mathbb{Z}^+$
37.	The additive identity of integer is $\dots\dots\dots$ A) 0                      B) 1                      C) 2                      D) 3
38.	$6 + (-10) = \dots\dots\dots$ A) 4                      B) 8                      C) -8                      D) -4
39.	$(-7) + 3 > \dots\dots\dots$ A) -5                      B) -4                      C) 2                      D) 0
40.	The additive neutral of integer is $\dots\dots\dots$ A) 0                      B) 1                      C) 2                      D) 3
41.	$3 + (-11) = \dots\dots\dots$ A) 4                      B) 8                      C) -8                      D) -4
42.	$8 + (-6) > \dots\dots\dots$ A) 2                      B) -4                      C) 3                      D) 5
43.	The additive inverse of $(-5)$ is $\dots\dots\dots$ A) 5                      B) 7                      C) 9                      D) 11

44.	The additive inverse of $(-7)$ is .....	A) 5	B) 7	C) 9	D) 11
45.	$-10 + 7 =$ .....	A) 3	B) 17	C) -3	D) -17
46.	$[8 + (-3)] \times (-3) =$ .....	A) -15	B) -4	C) 15	D) 0
47.	$-10 + 2 =$ .....	A) 3	B) -8	C) -3	D) -17
48.	The value of expression : $(-5) \times [7 + (-5)] =$ .....	A) -2	B) -10	C) 2	D) 10
49.	$7[6 + (-3)] =$ .....	A) -42	B) -21	C) 21	D) 18
50.	The additive inverse of 2 is .....	A) -2	B) -4	C) -6	D) -12
51.	$ -5  +$ ..... $= 0$	A) -5	B) -10	C) 0	D) 5
52.	The additive inverse of 4 is .....	A) -2	B) -4	C) -6	D) -12
53.	$ -7  +$ ..... $= 0$	A) -7	B) -14	C) 0	D) 7
54.	$ -2  +  2  =$ .....	A) 0	B) 2	C) 4	D) -10
55.	..... $+  -3  = 0$	A) -6	B) -3	C) 0	D) 3
56.	$ -7  +  7  =$ .....	A) 0	B) 14	C) 49	D) -10
57.	The additive inverse of 0 is .....	A) -2	B) -4	C) 0	D) -12
58.	$6 + (-6) =$ .....	A) 0	B) 12	C) 6	D) -6
59.	$5 + (-5) =$ .....	A) 5	B) 10	C) 0	D) -5

60.	$9 + (-9) = \dots\dots\dots$ A) 18                      B) 0                      C) 9                      D) -9
61.	$11 + (-11) = \dots\dots\dots$ A) 11                      B) 22                      C) 0                      D) -22
62.	The additive identity of integer is $\dots\dots\dots$ A) 0                      B) 1                      C) 2                      D) 3
63.	The additive inverse of $(-5)$ is $\dots\dots\dots$ A) 5                      B) 7                      C) 9                      D) 11
64.	The additive inverse of 2 is $\dots\dots\dots$ A) -2                      B) -4                      C) -6                      D) -12
65.	The additive inverse of 0 is $\dots\dots\dots$ A) -2                      B) -4                      C) 0                      D) -12
66.	$-5 + 15 = \dots\dots\dots$ A) -10                      B) 20                      C) 10                      D) -20
67.	$5 + (-9) = \dots\dots\dots$ A) 4                      B) 8                      C) -8                      D) -4
68.	$ -2  +  6  = \dots\dots\dots$ A) 12                      B) 2                      C) -2                      D) 8
69.	$ -9  - 11 = \dots\dots\dots$ A) 12                      B) -2                      C) 8                      D) -12
70.	$2 \times (-8) = \dots\dots\dots$ A) 18                      B) -18                      C) 10                      D) -16
71.	$(-4) \times 9 = \dots\dots\dots$ A) 12                      B) -36                      C) 36                      D) -10
72.	$(-5) \times (-6) = \dots\dots\dots$ A) 18                      B) -30                      C) 30                      D) -10
73.	$(-32) \div (-8) = \dots\dots\dots$ A) 4                      B) -4                      C) 40                      D) -24
74.	$(-18) \div 3 = \dots\dots\dots$ A) -9                      B) -3                      C) -2                      D) -6
75.	$24 \div (-4) = \dots\dots\dots$ A) -2                      B) -3                      C) -4                      D) -6

76.	If $X = 7$ , $y = -6$ then $XY =$ .....	A) 42	B) -42	C) 76	D) -76
77.	$-2 + 6 =$ .....	A) 4	B) 8	C) -8	D) -4
78.	$6 + (-2) =$ .....	A) 4	B) 8	C) -8	D) -4
79.	$-10 + 7 =$ .....	A) 3	B) 17	C) -3	D) -17
80.	$6 \times (-3) =$ .....	A) 18	B) -18	C) 10	D) -10
81.	$(-1) \times 5 =$ .....	A) 5	B) -12	C) 10	D) -5
82.	$(-2) \times (-9) =$ .....	A) 18	B) -18	C) 10	D) -10
83.	$(-12) \div (-2) =$ .....	A) 6	B) -6	C) 10	D) -10
84.	$(-18) \div 2 =$ .....	A) -9	B) -3	C) -2	D) -6
85.	$24 \div (-12) =$ .....	A) -2	B) -3	C) -4	D) -6
86.	$[8 + (-3)] \times (-3) =$ .....	A) -15	B) -4	C) 15	D) 0
87.	If $X = 8$ , $y = 9$ then $XY =$ .....	A) 72	B) -72	C) 89	D) -89
88.	The multiplicative identity of integer is .....	A) 0	B) 1	C) 2	D) 3
89.	$-3 + 9 =$ .....	A) -6	B) 12	C) 6	D) -12
90.	$6 + (-10) =$ .....	A) 4	B) 8	C) -8	D) -4
91.	$-10 + 2 =$ .....	A) 3	B) -8	C) -3	D) -17



92.	$5 \times (-1) = \dots\dots\dots$ A) 18                      B) -18                      C) 10                      D) -5
93.	$(-2) \times 6 = \dots\dots\dots$ A) 12                      B) -12                      C) 10                      D) -10
94.	$(-3) \times (-8) = \dots\dots\dots$ A) 18                      B) -24                      C) 24                      D) -10
95.	$(-8) \div (-4) = \dots\dots\dots$ A) 2                      B) -18                      C) 12                      D) -2
96.	The image of ( 5 , 1 ) by translation ( X - 1 , y + 2 ) is ..... A) ( 4 , 3 )                      B) ( 3 , 4 )                      C) ( 2 , 5 )                      D) ( 1 , 6 )
97.	The image of ( 3 , 1 ) by translation ( X + 1 , y + 2 ) is ..... A) ( 3 , 2 )                      B) ( 4 , 3 )                      C) ( 5 , 4 )                      D) ( 6 , 5 )
98.	The image of ( 3 , - 3 ) by translation ( - 1 , - 2 ) is ..... A) ( 4 , - 3 )                      B) ( 3 , - 4 )                      C) ( 2 , - 5 )                      D) ( 1 , - 6 )
99.	The image of ( 2 , 1 ) by translation ( - 1 , - 2 ) is ..... A) ( 4 , 2 )                      B) ( 3 , 1 )                      C) ( 2 , 0 )                      D) ( 1 , - 1 )
100.	The image of ( 5 , 1 ) by translation ( - 1 , 2 ) is ..... A) ( 4 , 3 )                      B) ( 3 , 4 )                      C) ( 2 , 5 )                      D) ( 1 , 6 )
101.	The image of the point ( 3 , 1 ) by the translation ( 1 , 2 ) is ..... A) ( 3 , 2 )                      B) ( 4 , 3 )                      C) ( 5 , 4 )                      D) ( 6 , 5 )
102.	If A ( - 2 , 2 ) and B ( - 5 , 2 ) , then AB = ..... Length units. A) 1                      B) 2                      C) 3                      D) 4
103.	If A ( 0 , 1 ) and B ( 0 , 5 ) , then AB = ..... Length units. A) 1                      B) 2                      C) 3                      D) 4
104.	The image of ( - 5 , - 4 ) by translation ( X - 1 , y - 2 ) is ..... A) ( - 3 , - 3 )                      B) ( - 4 , - 4 )                      C) ( - 5 , - 5 )                      D) ( - 6 , - 6 )
105.	The image of ( 5 , - 1 ) by translation ( X - 1 , y - 2 ) is ..... A) ( 4 , - 3 )                      B) ( 3 , - 4 )                      C) ( 2 , - 5 )                      D) ( 1 , - 6 )
106.	The image of ( 4 , 3 ) by translation ( X - 1 , y - 2 ) is ..... A) ( 4 , 2 )                      B) ( 3 , 1 )                      C) ( 2 , 0 )                      D) ( 1 , - 1 )
107.	The image of ( 4 , 2 ) by translation ( X + 1 , y + 2 ) is ..... A) ( 3 , 2 )                      B) ( 4 , 3 )                      C) ( 5 , 4 )                      D) ( 6 , 5 )

108.	The image of $(-5, -4)$ by translation $(-1, -2)$ is ..... A) $(-3, -3)$ B) $(-4, -4)$ C) $(-5, -5)$ D) $(-6, -6)$
109.	If A $(-2, 2)$ and B $(-3, 2)$ , then AB = ..... Length units. A) 1      B) 2      C) 3      D) 4
110.	If A $(0, 1)$ and B $(0, 3)$ , then AB = ..... Length units. A) 1      B) 2      C) 3      D) 4
111.	The image of $(-3, -2)$ by translation $(X-1, y-2)$ is ..... A) $(-3, -3)$ B) $(-4, -4)$ C) $(-5, -5)$ D) $(-6, -6)$
112.	The image of $(3, 2)$ by translation $(X-1, y-2)$ is ..... A) $(4, 2)$ B) $(3, 1)$ C) $(2, 0)$ D) $(1, -1)$
113.	The image of $(2, 4)$ by translation $(X-1, y+2)$ is ..... A) $(4, 3)$ B) $(3, 4)$ C) $(2, 5)$ D) $(1, 6)$
114.	The image of $(2, 0)$ by translation $(X+1, y+2)$ is ..... A) $(3, 2)$ B) $(4, 3)$ C) $(5, 4)$ D) $(6, 5)$
115.	The image of $(-3, -2)$ by translation $(-1, -2)$ is ..... A) $(-3, -3)$ B) $(-4, -4)$ C) $(-5, -5)$ D) $(-6, -6)$
116.	The image of $(3, 2)$ by translation $(-1, -2)$ is ..... A) $(4, 2)$ B) $(3, 1)$ C) $(2, 0)$ D) $(1, -1)$
117.	The image of $(2, 4)$ by translation $(-1, 2)$ is ..... A) $(4, 3)$ B) $(3, 4)$ C) $(2, 5)$ D) $(1, 6)$
118.	The image of $(-4, -3)$ by translation $(X-1, y-2)$ is ..... A) $(-3, -3)$ B) $(-4, -4)$ C) $(-5, -5)$ D) $(-6, -6)$
119.	The image of $(2, -4)$ by translation $(X-1, y-2)$ is ..... A) $(4, -3)$ B) $(3, -4)$ C) $(2, -5)$ D) $(1, -6)$
120.	The image of $(5, 4)$ by translation $(X-1, y-2)$ is ..... A) $(4, 2)$ B) $(3, 1)$ C) $(2, 0)$ D) $(1, -1)$
121.	The image of $(4, 2)$ by translation $(X-1, y+2)$ is ..... A) $(4, 3)$ B) $(3, 4)$ C) $(2, 5)$ D) $(1, 6)$
122.	The image of $(-4, -3)$ by translation $(-1, -2)$ is ..... A) $(-3, -3)$ B) $(-4, -4)$ C) $(-5, -5)$ D) $(-6, -6)$

**2 Answer the following :**

**1** Arrange the following numbers in an ascending order :

[a]  $|-9|$  ,  $2^2$  ,  $-5$  , zero and  $|7|$  "Suez - 2016"

[b]  $(-2)^3$  ,  $(-3)^2$  ,  $(-1)^{15}$  and  $(-5)^2$  "Obour - Kalyoubia - 2019"

**2** Use the properties of addition in  $\mathbb{Z}$  to find :

[a]  $125 + (-117) + (-125)$  (State the property used in each step)

"Beheira - 2017"

[b]  $37 + 25 + 63 + 75$

"Aswan - 2018"

**3** Use the distributive property to find the result of each of the following :

[a]  $6 \times [(-2) + (-7)]$  "Monofia - 2016"

[b]  $25 \times 9 + 25 - 25 \times 9$  "East Nars City - Cairo - 2019"

**4** In a Cartesian coordinates plane , locate the points B (2 , 1) , C (-2 , 1) , then find :

[a] The length of  $\overline{BC}$

[b] The image of  $\overline{BC}$  by translation (0 , -2) "Alexandria - 2017"

**5** Determine in the coordinates plane the position of the points A (1 , 1) , B (1 , 3) , then find the image of the line segment  $\overline{AB}$  by translation (-3 , 2)

"Fayoum - 2017"

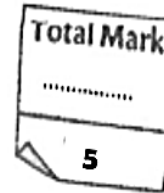
**6** On the coordinate plane :

Locate the points A (3 , -2) , B (1 , 1) and C (3 , 1) , then :

[a] Find the length of  $\overline{BC}$

[b] Draw the image of  $\triangle ABC$  by translation  $(x + 2 , y + 3)$

## Model 1



1 Choose the correct answer from the given ones :

(3 Marks)

1 The multiplicative identity element in  $\mathbb{Z}$  is .....

( - 1 or 1 or 0 or 2 )

2 If  $X(-2, 1)$  and  $Y(3, 1)$ , then the length of  $\overline{XY}$  = ..... length units.

( 0 or 1 or 3 or 5 )

3 The integer lies between - 1 and - 4 is .....

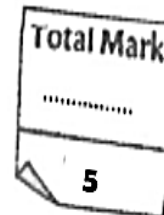
( - 5 or - 2 or 3 or 5 )

2 Use the properties of addition in  $\mathbb{Z}$  to find :

(2 Marks)

$$(-17) + 25 + 17$$

## Model 2



1 Choose the correct answer from the given ones :

(3 Marks)

1 The image of the point  $(-3, 2)$  by translation  $(x + 1, y)$  is .....

( (-2, 2) or (-2, 3) or (-2, -2) or (2, 2) )

2 If  $a \in \{2, -5, -3\} \cap \{5, -2, -3\}$ , then  $a =$  .....

( - 3 or 2 or 5 or - 5 )

3  $\mathbb{Z} - \mathbb{N} =$  .....

(  $\mathbb{Z}^+$  or  $\mathbb{Z}^-$  or  $\emptyset$  or  $\{0\}$  )

2 Use the distributive property to find the result :  $25 \times 9 + 25 - 25 \times 9$  (2 Marks)

## ***Model Answer***

- 1 ) b 2) a 3) b 4) b 5) a 6) b  
7 ) b 8) a 9) c 10) b 11) c 12) a  
13) b 14) a 15) d 16) b 17) a 18) b  
19) a 20) a 21) b 22) a 23) a 24) d  
25) c 26) c 27) a 28) c 29) c 30) c  
31) c 32) c 33) b 34) c 35) b 36) a  
37) a 38) d 39) a 40) a 41) c 42) a  
43) a 44) b 45) c 46) a 47) b 48) b  
49) c 50) a 51) a 52) b 53) a 54) c  
55) b 56) b 57) c 58) a 59) c 60) b  
61) c 62) a 63) a 64) a 65) c 66) c  
67) d 68) d 69) b 70) d 71) b 72) c  
73) a 74) d 75) d 76) b 77) a 78) a  
79) c 80) b 81) d 82) a 83) a 84) a  
85) a 86) a 87) a 88) b 89) c 90) d  
91) b 92) d 93) b 94) c 95) a 96) a  
97) b 98) c 99) d 100) a 101) b 102) c  
103) d 104) d 105) a 106) b 107) c 108) d  
109) a 110) b 111) b 112) c 113) d 114) a  
115) b 116) c 117) d 118) c 119) d 120) a  
121) b 122) c